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ATOMIC HYDROGEN MASER*

H. M. Goldenberg,† D. Kleppner, and N. F. Ramsey
Harvard University, Cambridge, Massachusetts
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Up to the present it has not been possible to produce maser oscillations with gaseous atoms due to the weakness of the magnetic dipole radiation matrix elements. However, with sufficiently long interaction times with the radiation field, oscillation can be achieved. Such increased interaction time and correspondingly narrowed resonance widths have been obtained in the present experiment by retaining the atoms in a storage box[1,2] with suitable walls. In this fashion self-sustained emission at the atomic hydrogen hyperfine transition frequency has been observed in an atomic beam maser. Mean interaction times up to 0.3 sec have so far been measured.

A diagram of the apparatus is given in Fig. 1. Atomic hydrogen from a Wood's discharge source passes through a six-pole state selecting magnet which focuses atoms in the $F=1$, $m=0$ and $F=1$, $m=1$ states onto an aperture in a paraffin-coated quartz bulb. The bulb is located in the center of a cylindrical rf cavity, operating in the TE_{011} mode, which is tuned to the ($F=1$, $m=0 \rightarrow F=0$, $m=0$) hyperfine transition frequency, approximately 1420.405 Mc/sec. The conditions are such that the atoms spontaneously radiate to the lower hyperfine level while in the bulb. The atoms make random collisions with the paraffin-covered bulb wall and eventually leave the bulb through the entrance aperture. Due to the small interaction with the paraffin surface they are not seriously perturbed for at least 10^4 collisions.

The criterion for self-sustained oscillation is that the power delivered to the cavity by the atomic beam must equal that lost by the cavity. In our case, with an average interaction time of 0.3 sec and a loaded cavity $Q=60\,000$, the minimum beam flux needed for oscillation is 4×10^{12} particles per seco..d. The maximum power delivered to the cavity by this beam is approximately 10^{-12} watt. However, in the present case the detecting probe is weakly coupled to the cavity so that the detected power is considerably less. The width of the resonance $\Delta\nu_\gamma$ without maser operation is about 1 cps as compared to several kc/sec in a conventional ammonia maser. The bulb used is approximately 15 cm in diameter, and has an entrance aperture of 1-mm radius. It is loaded with paraffin[3] in air, and is then baked while under vacuum.

The signal is detected by mixing it with a 1450-

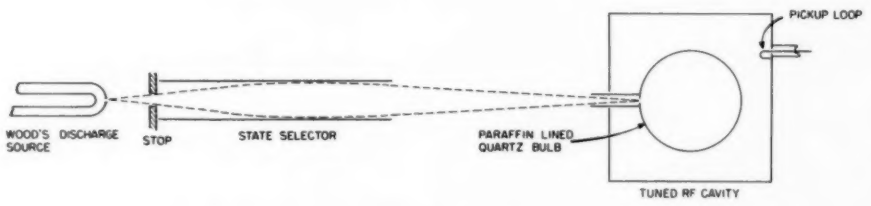

FIG. 1. Schematic diagram of atomic hydrogen maser.

Mc/sec signal obtained from the 5-Mc/sec output from an Atomichron by multiplication in a Gertsch FM4A frequency multiplier. The 29.595-Mc/sec i.f. signal is amplified and mixed in a phase discriminator with a similar signal from a Gertsch AM-1 stabilized oscillator. The resultant low-frequency signal passes through an *RC* filter, which limits the detector's over-all bandwidth, and is displayed on an oscilloscope.

In addition to oscillation, stimulated emission has also been observed. In this case oscillation is completely suppressed by shortening the mean time in the bulb (by means of a larger entrance aperture) or by loading the cavity. A pulse of rf at the transition frequency is fed into the cavity by a second coupling loop (not shown in Fig. 1). If this pulse is of the correct intensity the electron spin of the atom is precessed 90°, and the atom will continue to radiate after the rf has been turned off. A picture of the oscilloscope trace of this radiation is shown in Fig. 2. The amplifier is gated off during the stimulating pulse at the beginning of the sweep. The signal envelope decays with a time constant equal to the mean time in the bulb. The oscillating component of the signal is due to a slight offset in the local oscillator which is introduced for purposes of display. An interesting feature of this method of inducing emission is that those atoms which happen to be in the cavity for greater than the mean time are subject to more than one rf pulse. The mean interaction time of these atoms is then longer than the mean time in the cavity, and is characterized by the total time between the pulses they experience. If the pulse intensity is reduced so that it takes two pulses to put an atom in a state of maximum radiation, then the resonance line has a width characteristic of the pulse repetition frequency, which can be made several times smaller than the natural linewidth in the bulb.

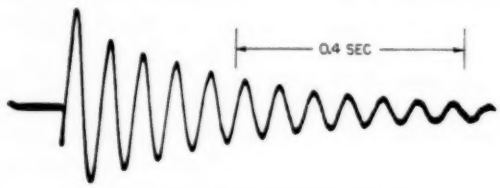

FIG. 2. Pulsed stimulated emission.

The magnetic-field-independent hyperfine transition has been used to observe the field-dependent Zeeman transitions, $F=1,\ m=0 \rightarrow F=1,\ m=1$) and ($F=1,\ m=0,\ \rightarrow F=1,\ m=-1$) by means of a double resonance method. A signal at the Zeeman frequency was coupled into the cavity at the same time that the hyperfine transition was occurring. The intensity of the hyperfine signal then decreased because the mean lifetime of atoms in the $F=1$, $m=0$ state was reduce due to their making transitions to the other $F=1$ states. This technique offers a method of measuring the static magnetic field to high accuracy.

This experiment represents the first successful use of the storage box principle. We believe that the atomic hydrogen maser will allow determination of the hyperfine splitting of the hydrogen isotopes to considerably higher precision than is now known. In addition, as a time standard, the maser may have a stability greatly exceeding that of any other standard yet proposed.

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†National Science Foundation Predoctoral Fellow.

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[2]D. Kleppner, N. F. Ramsey, and P. Fjelstad, Phys. Rev. Letters 1, 232 (1958).

[3]The paraffin used is a high melting point variety supplied under the trade name "Paraflint" by Moore and Munger, New York, New York.

THERMAL EXPANSION COEFFICIENT OF SOLID He^3†

Martin J. Klein and Raymond D. Mountain
Department of Physics, Case Institute of Technology, Cleveland, Ohio
(Received September 16, 1960)

Bernardes and Primakoff[1] have recently calculated many of the properties of solid He^3 on the basis of a theoretical model whose essential feature is the presence of a "symmetry" or "exchange" interaction between nearest neighbor atoms. They predict that at pressures comparable to the melting pressure solid He^3 is a nuclear antiferromagnet with a Curie temperature of about 0.1°K, but that at pressures in excess of 130 atmospheres or so the solid He^3 should become a nuclear ferromagnet. The purpose of this note is to point out that if this transition to ferromagnetic behavior occurs at high pressures, then it should be accompanied by a change in the sign of the coefficient of thermal expansion. Since Bernardes and Primakoff have shown that the thermal expansion coefficient is negative at pressures of the order of the melting pressure for temperatures less than several tenths of a degree,[2] this means that one can expect a positive thermal expansion coefficient at the pressures ($\gtrsim$130 atm) and temperatures ($\lesssim$ 0.1°K) where the solid is ferromagnetic.

The argument leading to this conclusion is quite straightforward. The isobaric coefficient of thermal expansion, α, is defined as $V^{-1}(\partial V/T)_P$, and it can be determined with the help of the Maxwell relation which requires that $(\partial V/\partial T)_P = -(\partial S/\partial P)_T$. Now the pressure dependence of the entropy comes entirely from the fact that the exchange integral J depends on pressure, at least at temperatures of the order of 0.1°K or less where the lattice entropy is negligible compared to the entropy of the nuclear spins. All that is needed then is the pressure derivative of the exchange integral J, and Bernardes and Primakoff have already given this in the form[3]

$$T_c^{-1}(dT_c/dP) = J^{-1}(dJ/dP) \cong -1\times10^{-2}/\text{atm}. \quad (1)$$

Since J is negative at the melting pressure, Eq. (1) indicates that J will become positive when the pressure is increased by about 100 atm above the melting pressure. The derivative (dJ/dP) can be expected to have constant (positive) sign.

We may now see that the sign of α is reversed when J changes sign since[4] the pressure derivative of the entropy, at temperatures below T, is proportional to $(-)J^{-1}(dJ/dP)$. Hence α will be positive when J becomes positive. This can be seen more physically by the following argument. We can write $(\partial S/\partial P)_T$ as $(\partial S/\partial J)_T \times (dJ/dP)$. Following Bernardes and Primakoff we take (dJ/dP) as positive. The derivative $(\partial S/\partial J)_T$ will be negative for a ferromagnet when an increase in J increases the ordering of the spins, but it will be positive for an antiferromagnet, where an increase in J (i.e., a positive increment to J which is itself negative) decreases the ordering of the spins.

We should like to thank Professor Henry Primakoff for prepublication copies of his paper and for a helpful letter.

†This work has been supported in part by the National Carbon Company, a division of Union Carbide Corporation, and in part by the U. S. Atomic Energy Commission.

[1]N. Bernardes and H. Primakoff, Phys. Rev. 119, 968 (1960).

[2]Experimentally confirmed by S. G. Sydoriak, R. L. Mills, and E. R. Grilly, Phys. Rev. Letters 4, 495 (1960).

[3]Reference 1, Eq. (23), p. 973.

[4]Reference 1, Eq. (51), p. 976.

TEMPERATURE DEPENDENCE OF THE INTERNAL FIELD IN FERROMAGNETS*

D. E. Nagle, H. Frauenfelder,† R. D. Taylor, D. R. F. Cochran, and B. T. Matthias‡
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico
(Received September 22, 1960)

The saturation magnetization of a ferromagnet varies with temperature in a characteristic and well-investigated manner; it reaches a limiting value at temperatures well below the Curie temperature T_C and vanishes at and above T_C. The behavior of the internal magnetic field, however, is much less well known. Because the theories of internal fields in ferromagnets are still far from satisfactory,[1] accurate and detailed measurements of these internal fields over a wide temperature range are important; the effects near the Curie point are particularly revealing. Preliminary work on such problems has been discussed by the Argonne group.[1] In the present Letter, we report a determination of the temperature variation of the internal magnetic field in Fe ($T_C = 1043°K$) and in a CoPd alloy ($T_C = 275°K$) by means of the Mössbauer effect.

That the Mössbauer effect is well suited for the investigation of internal fields no longer needs any justification.[1] The radioisotope Fe^{57} is very convenient for such experiments, since Fe itself is a ferromagnet and since the Mössbauer spectrum of its 14.4-kev gamma ray is well known.[2] The emission spectrum of an Fe source at room temperature consists of six lines, well separated by the Zeeman effect due to the internal magnetic field. With an Fe absorber of identical internal field, these lines give rise to a Mössbauer spectrum consisting of a prominent central absorption line and five strong satellites on either side, two of them doublets. The internal field has been deduced from the splitting of these lines.[2]

Complications arise when the source and the absorber have different internal fields. The six emission and six absorption lines of different spacing then give rise to as many as 36 lines; the Mössbauer spectrum becomes harder to find and harder to identify. There are three ways to circumvent this difficulty. One can employ a source having no effective internal field where the six emission lines are collapsed into one,[3] one can utilize a single line absorber,[4,5] or one can reduce the number of lines by selecting plane[2] or circularly[6] polarized gamma rays. The best method will depend on the particular problem. We have chosen the first approach for Fe and the third one, with selection of circularly polarized gamma rays, for CoPd.

A CoPd source was prepared by electroplating Co^{57} onto a CoPd alloy (92% Pd, 8% Co) and heating the CoPd foil in a vacuum furnace at 1000°C for two hours. The source was then placed in a cryostat which allowed the source temperature to be varied from 88°K to room temperature. The Mössbauer spectrum was observed with an Fe absorber, enriched to 75% Fe^{57} and of equivalent thickness 2.2 mg/cm². This absorber was mounted on a Jensen 8-inch Flexair woofer and moved sinusoidally at a frequency of 11 sec^{-1}. The output pulses from a scintillation counter were energy selected by a single-channel analyzer and modulated with a saw tooth voltage, which was locked in with the speaker drive. The modulated pulses were displayed on a 400-channel RIDL analyzer and thus yielded directly the desired Mössbauer spectrum,[4] as shown in Fig. 1. Due to the sinusoidal drive and linear display, the velocity scale in Fig. 1 is sinusoidal. The slight drop to the right in each spectrum is caused by dead-time effect in the 400-channel analyzer.

A series of measurements was taken in which the temperature of the CoPd source was varied and the temperature of the Fe absorber was 24°C.

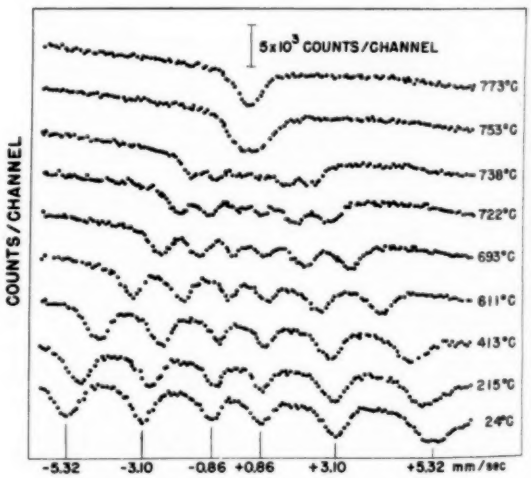

FIG. 1. Mössbauer spectra of a CoPd source at 24°C and an Fe absorber as functions of the absorber temperature. Positive velocity is taken to mean source moving away from absorber.

When the CoPd was above its Curie point of 275°K, the six-line spectrum was obtained, similar to the lowest curve of Fig. 1; such a spectrum is typical of a single-line source with an Fe absorber. Thus CoPd above its Curie point possesses only a very small effective magnetic field: From the position of the lines and from the line widths, an upper limit of 2000 oersteds is obtained. Below the Curie point, the splitting pattern changes rapidly, indicating the appearance of an internal magnetic field. At 88°K, astonishingly enough, the pattern is typical of source and absorber with identical fields. Hence at 88°K, the field at the Fe^{57} nucleus in CoPd is 3.3×10^5 oersteds,[2] the same as that of Fe^{57} in Fe at temperatures well below the Curie point. Details of these measurements will be published elsewhere.[7]

The single emission line of the CoPd source above its Curie point now offers a convenient tool for the investigation of the internal field in Fe. For this experiment, an absorber (5 mg/cm² Fe^{57}, 75% enriched) was placed in a furnace equipped with thin entrance and exit windows and containing an atmosphere of hydrogen. The CoPd source was mounted on the speaker and the transmission spectrum recorded as a function of the absorber temperature. Some typical spectra obtained in this way are shown in Fig. 1.

The curves in Fig. 1 show the decrease in the internal magnetic field, the temperature shift[8,9] of the center of the spectrum, the decrease in Mössbauer absorption due to the Debye-Waller factor, and finally the disappearance of the effective magnetic field in the Fe absorber at the Curie point.

The relative magnetic field at the Fe^{57} nucleus, as deduced from the curves in Fig. 1 and some additional data, is plotted in Fig. 2 as a function of T/T_C. For comparison, the relative saturation magnetization[10] is indicated by the solid line.

We thank Dr. R. M. Bozorth for determining the Curie point of our CoPd alloy, Dr. C. E. Olsen for the preparation of the CoPd alloy and the annealing of the source, and Dr. W. E. Keller for the loan of his DYNA amplifier. We are grateful to Dr. P. P. Craig and Dr. J. G. Dash for stimulating discussions and to Mr. R. Hanft for his tireless efforts during the experiment.

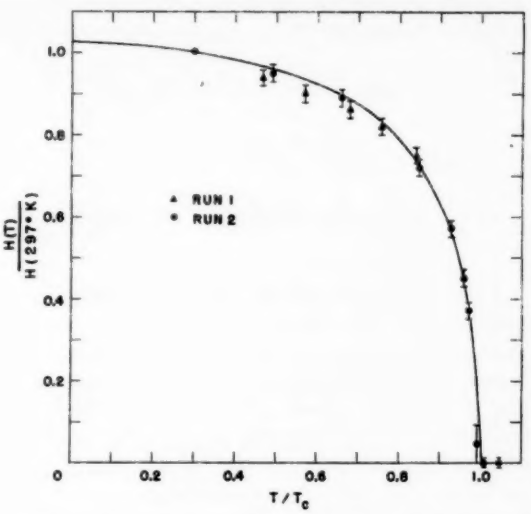

FIG. 2. Relative internal magnetic field $H(T)/H(297°K)$ at the Fe^{57} nuclei in an Fe absorber, as deduced by Mössbauer effect. The solid line indicates the relative saturation magnetization, $\sigma(T)/\sigma(297°K)$, of Fe.

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†Consultant, University of Illinois, Urbana, Illinois.

‡Consultant, Bell Telephone Laboratories, Murray Hill, New Jersey.

[1]Mössbauer Effect, Allerton House Conference, edited by H. Frauenfelder and H. Lustig (University of Illinois, Urbana, 1960).

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[6]H. Frauenfelder, D. E. Nagle, R. D. Taylor, D. R. F. Cochran, and W. M. Visscher (to be published).

[7]R. D. Taylor, D. E. Nagle, H. Frauenfelder, and D. R. F. Cochran (to be published).

[8]R. V. Pound and G. A. Rebka, Jr., Phys. Rev. Letters 4, 274 (1960).

[9]B. D. Josephson, Phys. Rev. Letters 4, 341 (1960).

[10]American Institute of Physics Handbook (McGraw-Hill Book Company, New York, 1957), p. 5-208.

EQUIVALENCE OF THE CRITICAL CONCENTRATIONS IN THE ISING AND HEISENBERG MODELS OF FERROMAGNETISM

R. J. Elliott and B. R. Heap
Clarendon Laboratory, Oxford, England

and

D. J. Morgan and G. S. Rushbrooke
King's College, Newcastle upon Tyne, England
(Received September 19, 1960)

If a magnetic crystal is diluted but otherwise unchanged by the addition of nonmagnetic atoms randomly arranged in the lattice, the transition temperature falls. There is a certain concentration of magnetic atoms below which the system will not exhibit a transition to magnetic order. At this critical concentration the transition temperature is at 0°K. It is the purpose of this Letter to show that in ferromagnetic crystals the critical concentration is a topological property of the lattice and is the same in the Heisenberg and Ising models and is independent of the value of the spin S. Nearest neighbor interactions only are assumed so that the Hamiltonians are $-2J\sum_{\langle i,j\rangle}\vec{S}_i\cdot\vec{S}_j$ and $-2J\sum_{\langle i,j\rangle}S_i^{\,z}S_j^{\,z}$ for the Heisenberg and Ising models, respectively.

The result may be derived from a consideration of the behavior of the susceptibility χ when expanded in a power series in the concentration p of magnetic atoms. In the random dilute system let there be N_1 isolated magnetic atoms, N_2 isolated nearest neighbor pairs, ... N_n isolated connected clusters of n magnetic atoms. For $n>2$ there will be various types of cluster which are specified by α, say. N_n is therefore subdivided into the numbers $N_{n,\alpha}$ of each type. The susceptibility can be written as

$$\chi kT/4\beta^2=\sum_{n,\alpha}N_{n,\alpha}\langle[S^z(n,\alpha)]^2\rangle_T, \tag{1}$$

where the last bracket is the thermal average of the square of the z component of the total spin of the cluster. At any T this can be evaluated only if the energy levels of the cluster are evaluated, but at $T=0$°K it can be simply written down for any cluster.

In the Ising problem the lowest energy state has all the spins oriented in the same direction and

$$\langle[S^z(n,\alpha)]^2\rangle_0=n^2S^2. \tag{2}$$

In the Heisenberg problem the lowest state is the symmetric one of maximum total spin nS and

$$\langle[S^z(n,\alpha)]^2\rangle_0=\tfrac{1}{3}nS(nS+1). \tag{3}$$

The $N_{n,\alpha}$ can be expanded in powers of p, each one having p^n as its smallest power. Equation (1) becomes, using (2),

$$\sum_n N_n n^2S^2=NS^2\{p+a_2p^2+a_3p^3+\ldots\}, \tag{4}$$

and using (3), together with the relation $\sum_n nN_n=Np$,

$$\tfrac{1}{3}\sum N_n(nS+n^2S^2)=NS^2\left\{\frac{S+1}{S}p+a_2p^2+a_3p^3\ldots\right\}. \tag{5}$$

The a's depend only on the topology of the lattice and are the same in (4) and (5). Thus χ diverges at the same value of p in both models. The $N_{n,\alpha}$ and the a_n have been evaluated up to and including $n=5$. The critical concentration p_c has been evaluated by estimating the radius of convergence from the sequence $|a_n|^{-1/n}$ and by inverting the series. The inverted series was then truncated after 2, 3, 4, and 5 terms and equated to zero to obtain a sequence of approximations. The resulting values of p_c are as follows and the convergence of the method leads us to believe that they are accurate to within 10%. Simple cubic: 0.28; body-centered cubic: 0.22; face-centered cubic: 0.18; plane square: 0.48; plane honeycomb: 0.49; triangular net: 0.36. The finite value of p_c predicted for plane lattices in the Heisenberg model is at first sight in contradiction with the spin wave theory which predicts no ordering; although they can be reconciled if $T_c=0$, $p_c\leq p\leq 1$. Series methods,[1] however, seem to indicate a finite T_c for these lattices, so that unexplained discrepancies remain.

These results may be compared with those obtained by other methods. Brout[2] makes a more sophisticated attack on the problem but the simplest approximation which he actually evaluates is only equivalent to considering the first two terms of the series. Elliott[3] has evaluated p_c in the constant-coupling approximation, but this

also places undue emphasis on the first two terms in the series[4] (5) and leads to a dependence of p_c on S which is a spurious result of the approximation.

The above theorem holds if more than next nearest neighbor interactions are included provided they are all ferromagnetic. It does not hold in antiferromagnetics[5] where the particular case of MnF_2 and CoF_2 mixed with ZnF_2 has been considered for comparison with the experimental results of Baker and Lourens.[6] The method of series expansion in p may be extended to finite T to obtain the variation of T_c with p.[4,5]

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[5]R. J. Elliott and B. R. Heap (to be published).
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INTERACTION OF HIGH-ENERGY PHONONS IN GERMANIUM*

Gianni Ascarelli
Physics Department, University of Illinois, Urbana, Illinois
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Many authors have investigated the non-Ohmic conductivity of germanium.[1-6] The theoretical explanation advanced by Shockley[7] to explain the experimental results of Ryder and others was that electrons which have been very much accelerated by the applied electric field will emit phonons and lose an appreciable fraction of their energy. Recently there have been attempts to observe these high-energy phonons. Aigrain[8] tried to detect them by means of the momentum they would transfer to electrons in a region of the sample where the electrons are "cold"; Hubner and Shockley[9] succeeded in an experiment similar to that attempted by Aigrain and observed such a transmitted phonon drag in silicon. We will report an experiment in which a transport of energy by phonons excited by hot electrons is observed.

In our experiment, a short voltage pulse produced low-temperature breakdown in the portion AB of a very pure n-type antimony-doped germanium crystal ($N_D - N_A \sim 4\times10^{13}$ cm^{-3}, $N_A \sim 5\times10^{11}$ cm^{-3}) placed outside an S-band microwave cavity (Fig. 1). A dc voltage supplied by a battery could be applied to the portion BC of the sample that was in the microwave cavity. The sample had its axis along the [111] direction.

At 4.2°K when the germanium sample was at thermal equilibrium with the liquid helium surrounding it, the unloaded Q of the cavity was of the order of 4000. Losses were principally due to the metal walls. (Losses originating from the sample would result in a value of Q of about 10^6.)

The cavity was placed on one of the symmetrical arms of a hybrid Tee; on the other there was a load with an impedance selected so that no power fed into the H arm could be detected out of the E arm. The detector used was a superheterodyne system. By means of such an arrangement, small changes in the properties of the cavity can be easily detected: Weber[10] was able to measure frequency shifts of 10^{-5}%; and we could easily

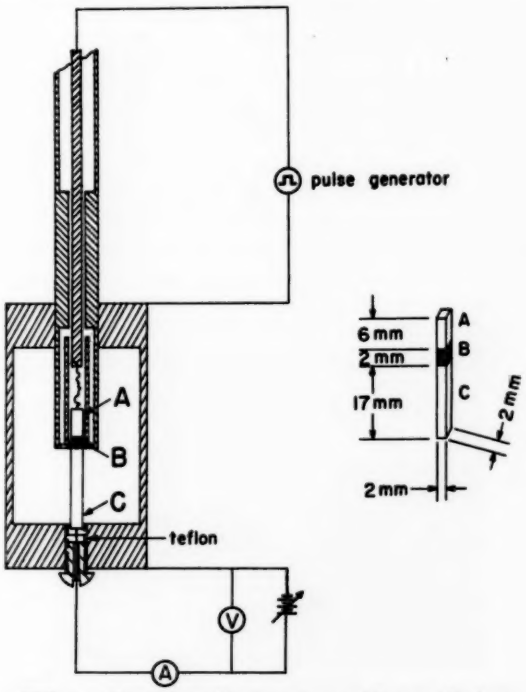

FIG. 1. Experimental arrangement for producing and detecting the "phonon beam." The portion of the sample in the vicinity of B is covered with tin. The cross section of the sample is 2×2 mm^2; all the contacts are made with tin.

detect changes in Q of 0.1%.

When a short pulse (e.g., 1.5 μsec, 30 v/cm) is applied to the portion AB of the sample it produces breakdown only in that portion which is outside the cavity. The microwave bridge is unbalanced for a short period of time corresponding to a change of the Q of the cavity. The unbalanced pulse out of the E arm is delayed approximately 0.5 μsec from the time of application of the breakdown pulse. The magnitude of the unbalance of the microwave bridge depends on the polarity of the pulse applied to AB: there is a larger unbalance when the drift velocity of the electrons is directed from A towards B rather than vice versa (Fig. 2). The application of a dc field (below breakdown) across BC does not affect the unbalance pulse whatever is the polarity of C with respect to B. The decay time constant agrees with what has been measured separately,[11] in similar conditions, when the decay of the electron density in portion AB is limited by thermal effects: $\tau_b = 0.45$ μsec and is determined by sample size. The propagation velocity of the perturbation detected in BC (measured by the delay of the microwave unbalance pulse with respect to the breakdown pulse) is of the order of 10^6 cm/sec.

From the photographs of oscilloscope traces reproduced in Fig. 2, it is apparent that the losses in the cavity when the drift velocity of the electrons is directed from A to B, are four times larger than when the drift velocity is oppositely directed. An order of magnitude can be given for the change in conductivity by considering the sensitivity of the apparatus and the geometry of the sample: it corresponds to a change of a factor of 5 of the conductivity of BC.

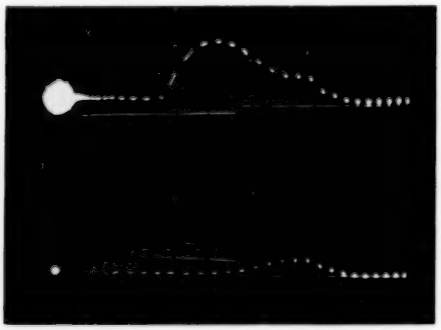

FIG. 2. Power output from the E arms of the hybrid Tee when a negative (upper trace) or positive (lower trace) pulse are applied to AB. The full line corresponds to the breakdown pulse applied to A and the dots correspond to a 10-Mc/sec sine wave applied to the z axis of the oscilloscope.

It was also noticed that if the portion AB of the sample is very short, so that the lines of force diverge appreciably, the magnitude of the unbalance pulse decreases very much.

The phenomenon is completely changed if the contacts at either A or B are slightly non-Ohmic. In this case a larger unbalance might appear when the drift velocity of the electrons is directed from B towards A rather than vice versa; this is particularly true if the contact at B is non-Ohmic. In these conditions a small dc field applied to BC will affect very much the unbalance pulse both in magnitude and in its time delay with respect to the breakdown pulse.

The explanation proposed for the observed phenomenon, in the case of Ohmic contacts, is that during breakdown of the portion AB of the sample, a beam of phonons is produced having a direction of propagation parallel to the drift velocity of the electrons. The portion BC of the sample acts as a sensitive bolometer: from the change in its conductivity we can evaluate the corresponding change in temperature produced by the phonon beam.

From the observed behavior, and by comparison with the measurements of thermal conductivity, we can attempt to evaluate an average phonon energy and an average phonon mean free path.

After the sample has been broken down, the electron gas will have its mean energy increased. Many of the collisions of the electrons will not ionize the remaining neutral donors but will instead produce phonons. As an example, an electron whose energy is 10^{-2} ev can lose up to 1.7×10^{-3} ev to a phonon.

In order of decreasing probability we can envisage three mechanisms by means of which the electron density in BC is increased:

(a) The phonons interact with the lattice, e.g., due to point imperfections (e.g., isotopes[12]) or by means of Umklapp processes.

(b) They interact with the electrons bound to the impurities. They could excite these electrons from the ground state to the level that has been split off from it by the valley-orbit interaction[13] ($\sim4\times10^{-3}$ ev for As, $\sim3\times10^{-3}$ ev for P, and should be much smaller for Sb). The bound electrons would then transfer their energy to the free electrons.

(c) They interact with the free electrons, increasing their energy and thus the probability of impact ionization.

In the calculation below, we assumed that when a phonon reaches the surface of the sample it

will dissipate its energy in the helium bath. From the observed change in conductivity the temperature increase of the portion BC can be calculated:

$$\frac{\sigma}{\sigma_0} \cong \exp\left[\frac{\epsilon}{kT_0}\left(\frac{\Delta T}{T_0}\right)\right] \sim 5,$$

where ϵ is the binding energy of the impurities and $\Delta T \sim 0.25°K$ for $T = 4.2°K$ ($\epsilon = 9.6\times10^{-3}$ ev for Sb).

Assuming that the portion AB of the sample has $f(N_D - N_A)$ free electrons produced by breakdown ($f \leq 1$) and that the electron-phonon collision time, τ_{ep}, is approximately 10^{-10} sec, as measured from cyclotron resonance experiments,[14] a beam of $f(N_D - N_A)\tau_b/\tau_{ep}$ phonons with an average energy $h\nu$ will be produced in AB during the time τ_b (τ_b is the phonon mean free time limited by boundary scattering). The fraction of these phonons arriving in BC that will dissipate their energy in this portion of the sample rather than in the helium bath will be τ_b/τ_p, where τ_p is the phonon mean free time for scattering by a mechanism other than the sample surface ($\tau_p > \tau_b$).

Keesom and Pearlman[15] measured the lattice specific heats in Ge; we will use their value for calculating the mean free time for phonon-lattice interaction.

Denoting by C_v the specific heat, we can write

$$C_v(\Delta T) = (h\nu)f(N_D - N_A)(\tau_b/\tau_{ep})(\tau_b/\tau_p),$$

and we obtain $\tau_p/f h\nu = 10^{10}$ where C_v has been taken equal to 30 ergs/cm³ °K. Similar calculations for processes (b) and (c) give values larger by many orders of magnitude indicating an increase in the corresponding τ_p.

A possible choice of $h\nu$ is to impose the condition that the mean free path calculated from τ_p (assuming, e.g., longitudinal acoustic phonons) agrees with what can be calculated from the data of Geballe and Hull.[12] Slack[16] has shown that, at the temperature T_{max} at which the thermal conductivity has a maximum, the mean free path, determined by scattering by point defects for phonons whose energy in kT_{max}, is equal to 1.15 times the smallest sample dimension. Geballe and Hull[12] obtained a maximum in the thermal conductivity at 18°K in a sample whose smallest dimension was 1.3 mm; the phonon mean free path determined by point defect scattering should accordingly be $\lambda = 0.15(kT_{max}/h\nu)^4$. In the case of longitudinal acoustic phonons, this gives $f^{1/3}h\nu = 6.2\times10^{-4}$ ev and a mean free path for scattering by point imperfections of the order of 5.5 cm.

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CONTRIBUTION OF EXCHANGE POLARIZATION OF CORE ELECTRONS TO THE MAGNETIC FIELD AT THE NUCLEUS OF Fe

D. A. Goodings and V. Heine
Royal Society Mond Laboratory, Cambridge, England
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Recent experiments[1,2] on the hyperfine spectrum of Fe^{57} showed that the effective magnetic field at the nucleus in ferromagnetic iron is directed oppositely to the magnetization with a value of -333 kgauss. Because of the negative sign it was recognized[2] that the dominant contribution to H_{eff} must arise from the contact interaction of s electrons in the iron core. Results of calculations are reported here which confirm this view and give greater insight into the effect.

Following Marshall,[3] the $3d+4s$ "conduction" electrons give a positive contribution to H_{eff}. About 50 kgauss come from some unquenched orbital motion due to spin-orbit coupling. There is also some exchange polarization of the "4s band" and some admixture of 4s-like functions into the "3d band." Together they are equivalent to having about 0.04 to 0.15 unpaired 4s electrons per atom, giving an H_{eff} of 70 to 250 kgauss. Thus the core electrons must contribute an H_{eff} of at least -450 kgauss to account for the experimental result.

Let the majority of 3d electrons have spin $\uparrow$. Then since exchange acts only between electrons of parallel spin, a core electron with spin $\uparrow$ experiences a stronger exchange force than the corresponding core electron with spin $\downarrow$. If the Hartree-Fock formalism is modified to allow such pairs of electrons to have different radial functions,[4] one can calculate $\psi_{nl\uparrow}(\vec{r})$ and $\psi_{nl\downarrow}(\vec{r})$ for each nl pair in a self-consistent manner. This is known as the unrestricted Hartree-Fock (UHF) method. In particular one obtains nonzero values for the quantities

$$\rho_{ns} = |\psi_{ns\uparrow}(0)|^2 - |\psi_{ns\downarrow}(0)|^2,$$

giving a contribution to H_{eff} from the contact interaction

$$H_{\text{core}} = (8\pi/3)\mu_B(\bar{S}/S)\textstyle\sum_{\text{core}}\rho_{ns}.$$

Here $\bar{S}$ is the effective spin from the measured net magnetic moment per atom, and S is the spin of the configuration for which the ρ_{ns} are calculated.

A UHF calculation was carried out by numerical methods[5] for the $(3d)^6(4s)^2\ {}^5D_4$ ground term of the free iron atom. The results, given in the first line of Table I, exemplify features found generally in exchange polarization of core electrons.[4,6] The additional exchange force from one or more unpaired p or d electrons tends to pull out $\psi_{ns\uparrow}(\vec{r})$ relative to $\psi_{ns\downarrow}(\vec{r})$ for inner ns levels, giving a large negative contribution to H_{core}. The opposite is true for outer levels. For example, in the free iron atom, the calculated value of ρ_{4s} was +1.02 which almost completely cancels the negative core contribution. In the metal, however, the contribution of the 4s electrons is expected to be much smaller in accordance with the estimates already given above. We note that the value $H_{\text{core}} = -355$ kgauss is not quite enough to explain the experimental result.

Another factor must be considered, namely that the 3d electrons may be more spread out in the metal than in the free atom.[7,8] The calculation was therefore repeated with 3d functions artificially expanded by about 5% at the maxima and 10% over the tail regions, and the results are shown in the lower line of Table I. The expanded 3d functions make the 3s electrons behave more like an "inner" shell, with the net result that H_{core} has a larger negative value. We conclude that it seems difficult to explain the large negative H_{eff} in the metal without assuming that the

Table I. Results of the UHF calculations. The ρ's are given in units of (Bohr radius)$^{-3}$.

	ρ_{1s}	ρ_{2s}	ρ_{3s}	$\sum_{\text{core}}\rho_{ns}$	H_{core}(kgauss)
Free atom	-0.03	-2.41	1.22	-1.22	-355
Expanded 3d functions	0.00	-2.10	0.67	-1.44	-420

3*d* electrons are 5 to 10% more spread out than in the free ion and that the amount of 4*s* admixed into the 3*d* band is rather small, unless there is some new additional effect such as a negative polarization of the conduction band.[7]

Incidentally the calculations show that if iron were a paramagnetic metal it would have a negative Knight shift.[9] Thus there is no difficulty in principle of understanding the occurrence of negative Knight shifts in platinum[10] and manganese.[11]

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TOROIDAL ENERGY SURFACES IN CRYSTALS WITH WURTZITE SYMMETRY

R. C. Casella

International Business Machines Research Laboratory, Poughkeepsie, New York

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This note describes the results of an investigation which demonstrates the possibility of toroidal energy surfaces in wurtzite-type solids. The model leads to an anisotropy in the Hall effect, which, in principle, can be made as large as one pleases by lowering the sample temperature.

Based on an earlier symmetry analysis,[1] the shapes of the constant-energy surfaces in the vicinity of the conduction and valence band extrema have been obtained for several crystals having the wurtzite symmetry. Analyses[1-6] of data on the optical absorption in hexagonal CdS, ZnS, ZnO, and CdSe indicate that for these materials the extrema lie at (or very close to) $\vec{k}=0$, where $\vec{k}$ is the electron wave vector. Therefore, the dispersion of the single-particle energy $E(\vec{k})$ has been investigated only in the neighborhood of the origin of the Brillouin zone. Study of the optical data also indicates that the conduction band wave functions at $\vec{k}=0$ transform according to the representation Γ_7 of the point group C_{6v} (including the effects of spin-orbit interaction), and that the corresponding wave functions associated with the upper and lower "split-off" valence bands belong to the representations Γ_9 and Γ_7, respectively. Expanding $E(\vec{k})$ about its value at $\vec{k}=0$ to second order in the wave vector components by means of the well-known $\vec{k}\cdot\vec{p}$ perturbation technique, where $\vec{p}$ is the electron momentum, the following results were obtained: (1) The constant-energy surfaces for the upper (Γ_9) valence band are doubly degenerate ellipsoids of revolution about the k_z axis ($z \parallel c$). (2) For the Γ_7 conduction and valence bands, the possibility of finite slopes[7] in $E(\vec{k})$ at $\vec{k}=0$ together with the existence of a single rotational axis in wurtzite-type materials (the c axis) allows the possibility of toroidal energy surfaces. Henceforth, we shall confine our attention to the toroidal surfaces and the physical properties of wurtzite-type crystals which one might expect to result therefrom.

For the Γ_7 bands $E(\vec{k})$ has two roots ($\mp$) of the form

$$E_{\mp}(\vec{k}) = E_0(0) \mp Lk_{\perp} + Ak_{\perp}^2 + Ck_z^2, \qquad (1)$$

where

$$k_{\perp} = (k_x^2 + k_y^2)^{1/2}, \quad L = m^{-1}|(1|\pi_y|2)|,$$

$$A = (2m)^{-1} + \textstyle\sum_i (m^2\omega_{0i})^{-1}|(1|\pi_x|i)|^2, \qquad (2)$$

and C is given by the right side of Eq. (2) with π_z substituted for π_x. In the above expressions, m is the free-electron mass, $\vec{\pi}$ is the generalized

form of $\vec{p}$ in the $\vec{k}\cdot\vec{p}$ perturbation scheme when spin-orbit coupling is included,[8] $|1\rangle$ and $|2\rangle$ are the states belonging to Γ_7 with energy $E_0(0)$, $|i\rangle$ is an intermediate state with energy $E_i(0)$, and $\omega_{0i} = E_0(0) - E_i(0)$. We have set $\hbar = 1$, throughout. Letting $\beta = L/2A$ and choosing the zero of energy such that $E_0(0) = A\beta^2$, Eq. (1) becomes

$$E_{\mp}(\vec{k}) = A(k_{\perp} \mp \beta)^2 + Ck_z^2. \tag{3}$$

We assume that A and C are both positive for the conduction band and both negative for the valence band. For the purpose of illustration we shall discuss only electrons in the conduction band. Since, by definition, $k_{\perp} \geq 0$, the root E_+ does not exist for $E(\vec{k}) < A\beta^2$. In the range $0 < E(\vec{k}) \leq A\beta^2$, the equation $E_- =$ constant describes a toroidal surface with cylindrical symmetry about the k_z axis. The cross section of the surface with a plane containing the k_z axis consists of two ellipses. The minimal line, $E_- = 0$, is a circle in the plane $k_z = 0$. For $E(\vec{k}) \geq A\beta^2$ both roots must be taken into account.

We have investigated the consequences of toroidal energy surfaces for the following properties: (1) Density of states, dN/dE. For a crystal of unit volume,

$$(dN/dE) = (4\pi)^{-1}[\beta/(AC)^{1/2}] = \text{constant}, \tag{4}$$

for $A\beta^2 \geq E \geq 0$. For $E >> A\beta^2$, dN/dE is proportional to $E^{1/2}$. A closed-form expression for dN/dE in the intermediate energy range has been obtained, but is too unwieldy to merit presentation here. In terms of interband transitions, the effect on the optical absorption of the sharp leading edge of an otherwise essentially parabolic density-of-states curve might be expected to sharpen the absorption edge. The importance of the effect will depend upon the magnitude of $A\beta^2$ relative to the scale of photon energies in the absorption curves. The magnitude of $A\beta^2$ is discussed in a subsequent paragraph. What the effects of the toroidal surfaces are on the dynamical properties of excitons has not been ascertained. It must be noted, however, that Thomas and co-workers[5] have succeeded in giving an account of the dependence of the absorption coefficient on photon frequency at the absorption edge for CdS by assuming phonon-assisted excitonic transitions and spherical energy bands. (2) Cyclotron frequency, ω_C. With a magnetic field $\vec{H}$ applied parallel to the c axis, the tube mass[9] m^*, which is inversely proportional to ω_C, is given by the relation

$$m_{\mp}^* = (2A)^{-1} k_{\perp} (k_{\perp} \mp \beta)^{-1}. \tag{5}$$

Electrons in the band E_- with $k_{\perp} < \beta$ rotate in a sense opposite to that for electrons with $k_{\perp} > \beta$. For electrons in the band E_- with $k_{\perp} = \beta$, $\omega_C = 0$. These electrons have group velocities parallel to $\vec{H}$. Electrons in either band with $k_{\perp} >> \beta$ have masses which approach a constant value, $(2A)^{-1}$. This result is consistent with the fact that the energy surfaces resemble ellipsoids of revolution about the k_z axis for $E(\vec{k}) >> A\beta^2$, except in the immediate vicinity of the line $k_{\perp} = 0$. (3) Transport properties. The zero-field components, $\sigma_{xx}^{(0)}$ and $\sigma_{zz}^{(0)}$, of the conductivity tensor as well as the Hall coefficients, $R(H \perp c)$ and $R(H \parallel c)$, have been calculated using the Jones-Zener[10] weak-field expansion of the Boltzmann equation. Particularly simple results obtain under the conditions $\Lambda = (A\beta^2/k_B T) >> 1$ and $\tau =$ constant. In the foregoing, k_B is Boltzmann's constant, T is the absolute temperature, and τ is the relaxation time. Physically, the condition $\Lambda >> 1$ implies that most of the carriers are in the band E_-. In addition to introducing formal simplifications this condition justifies the neglect of interband scattering provided the predominant scattering mechanism is elastic. Because of the assumption of a constant relaxation time, our results are to be interpreted only as indicating a measure of the anisotropy introduced by the toroidal surfaces. Subject to the foregoing restrictions the results are

$$\sigma_{xx}^{(0)} = (Ne^2\tau)A = \sigma_{yy}^{(0)}, \quad \sigma_{zz}^{(0)} = (Ne^2\tau)(2C), \tag{6}$$

$$R(H \perp c) = -(Nec_L)^{-1},$$

$$R(H \parallel c) = -(Nec_L)^{-1}\{2(\Lambda/\pi)^{1/2} \exp(-\Lambda)\}. \tag{7}$$

In Eqs. (6) and (7), N is the number of electrons in the conduction band, e is the absolute value of the electronic charge, and c_L is the velocity of light. The expression for $\sigma_{zz}^{(0)}$ is equal to that which would obtain for an ellipsoid of revolution, as is to be expected by inspection of Eq. (3). The most striking feature of these results is the strong anisotropy exhibited by the Hall coefficients. Apart from a proportionality constant of order unity, the ratio $R(H \parallel c)/R(H \perp c)$ diminishes as $\Lambda^{1/2} \exp(-\Lambda)$ with decreasing temperature. It can be shown that the exponential factor in $R(H \parallel c)$ persists if τ is generalized such that τ^2 is any analytic function of E. Physically, the coefficient

$R(H \| c)$ is small because electrons with $k_\perp < \beta$ rotate in a sense opposite to that for electrons with $k_\perp > \beta$ and because ω_c tends to zero as $k_\perp$ approaches β.

For CdS Thomas et al.[5] have estimated an upper bound of the order of a few hundredths of an ev for the displacement of any energy minima below $E(0)$ in the conduction band. In the toroidal model this energy displacement equals $A\beta^2$. Setting $A\beta^2 = 0.04$ ev, one obtains $\Lambda \approx 6$ at $T = 77°K$. Thus, one might hope to observe a large anisotropy in the Hall effect, if the model is correct.

It should be emphasized that the model results from an approximation. Extension of the $\vec{k}\cdot\vec{p}$ perturbation calculation to higher orders will lower the symmetry of the surfaces to one of sixfold rather than continuous rotational invariance. Whether or not the corrections are important will depend upon the size of β and the proximity of other bands. For $A\beta^2 = 0.04$ ev and $A = (2m)^{-1}$, β is typically of the order of one-tenth the shortest distance in $\vec{k}$ space between the point $\vec{k} = 0$ and any of the reduced zone boundaries parallel to the k_z axis.

Added note. The toroidal model is a direct consequence of the spin-orbit interaction. The nature of $E(\vec{k})$ in the limit of zero spin-orbit coupling is described in the earlier work of Rashba,[11] who has recently given independent consideration to the effects of including the interaction.[12]

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NUCLEAR POLARIZATION IN He^3 GAS INDUCED BY OPTICAL PUMPING AND DIPOLAR EXCHANGE*

M. A. Bouchiat,† T. R. Carver,‡ and C. M. Varnum
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received September 26, 1960)

Although almost complete polarization of alkali metal nuclei can be produced by optical pumping utilizing a buffer gas and a filter to remove the D_2 resonance light,[1,2] the number of atoms polarized is necessarily very small and is limited by optical opacity of the sample vapor. The usual range of pressure is 10^{-7} to 10^{-3} mm for successful detection and production of optical polarization. The Overhauser nuclear polarization effect[3] involving dipolar interactions[4-8] between such an optically polarized atom and the nucleus of a suitable buffer gas should provide a mechanism for the transfer of polarization to a gas at considerably higher pressure than that of the alkali vapor. In other words, the Overhauser effect should work not by the dipole-coupled relaxation of a saturated paramagnetic impurity toward a polarized equilibrium, but by the relaxation of an optically polarized impurity toward a nearly depolarized equilibrium. We have observed this effect in He^3 gas used as the buffer gas for the optical pumping of natural rubidium vapor.

The nuclear magnetic relaxation time in a liquid or a monatomic gas depends inversely on both the square of the magnetic moment of the nucleus and the square of the moment of other spins causing the relaxation. If He^3 is contaminated with "dissolved" Rb atoms having a moment 10^3 times as great as the He^3, a concentration of more than 1 part in 10^6 would dominate the relaxation. The dipole interaction terms in-

volving the spin-flip operators I_+S_+ and I_-S_- for the nuclear and electronic spins, respectively, are known to dominate the relaxation process,[8] so that as the Rb atoms relax from the optically polarized condition they flip He^3 nuclei to a polarized condition. In our experiment a partial pressure of 10^{-3} mm of Rb^{85+87} vapor should polarize the 2.8-atmosphere He^3 sample with a characteristic time of between 10^5 and 10^6 seconds, comparable to the He^3 relaxation time at that pressure. In actual fact the relaxation time was found to be shortened by what is almost certainly wall relaxation. This competing relaxation process reduces the amount of He^3 polarization below the approximately 10% polarization of the Rb atoms, but we have observed enhancement of the nuclear polarization by a factor of 10^4 above the initial Boltzmann distribution of 10^{-8}.

The experimental technique was as follows: A 30-cm^3 Pyrex bulb was outgassed and treated with G.E. Dri-Film (dimethyl-dichloro silane), a treatment[9] which has been found to be effective in preventing the recombination of atomic hydrogen,[10] and in increasing the relaxation time of optically pumped vapors.[11] The bulb was again baked out in vacuum, and natural Rb was distilled into a side tube connecting with the bulb through a small re-entrant orifice so that an excess of metallic Rb would not be present in the bulb itself. He^3 was admitted at 2.8 atmospheres STP after purification with hot calcium and a discharge with titanium electrodes to remove contaminant O_2 and N_2, and the bulb was sealed off with a breakoff to reclaim the He^3. The entire sample was placed within another Pyrex tube to conduct hot air and maintain the sample at 140°C, at which point the Rb pressure is 3×10^{-3} mm.

The Rb was polarized by irradiating only with the D_1 (7947A) line from a 4-cm diameter rf-excited Rb lamp passed through a thin film D_2 interference filter, and subsequently through plastic sheet type polarizer and quarter-wave plate designed for near infrared use. The optical pumping was monitored by light transmission through the cell and the observation of the $\Delta M = 1$, $\Delta F = 0$ transitions excited by a small rf loop around the sample.

In order to measure the degree of nuclear polarization, the nuclear resonance of He^3 was observed at 400 kc/sec in a field of 123.3 gauss produced by a large corrected solenoid capable of a homogeneity of a few milligauss over the sample volume. The detection system consisted of a crystal oscillator, twin-T bridge, preamp, i.f. strip, and a 400-kc/sec lock-in amplifier which directly phase-or amplitude-detects the signal. This apparatus had sufficient sensitivity to display the proton resonance in a 5-atmosphere propane sample with a single sweep equivalent to a 90° pulse. To make possible the detection of He^3 resonance for calibration and comparison with optical polarization, the He samples could be observed after prior polarization in a 10 000-gauss field. In the case of the optically polarized sample, the 400-kc/sec nuclear magnetic resonance (NMR) coil was wound directly on the outer Pyrex heating tube in a Helmholtz pair arrangement so that the optical pumping radiation could pass through the sample along the axis of symmetry of the solenoid. The NMR signals obtained were sufficiently intense ($S:N = 1000$) so that observation of the long relaxation time could be made with successive almost nondestructive fast-passage sweeps by reducing the incident 400-kc/sec power to a very low level. The time constants involved were 1800, 2450, 1200, and 2250 seconds for polarization by Rb at 140°C, depolarization at 140°C, polarization at 27°C by large external magnetic field, and depolarization at 30°C, respectively. The nuclear polarization could be inverted and observed in either emission or absorption by simply changing the circular polarizer from left to right.

There appears to be no reason in principle why this technique cannot be used to produce polarization comparable to that induced in the metal vapor. The polarization is reduced, however, by other relaxation processes. We have investigated the relaxation time in a number of samples which do not contain Rb and are not above room temperature. The results are not very reproducible, but we have generally found that the relaxation time is proportional to the density of the He^3, that the relaxation time doubles in going from 300°K to 77°K, and that deliberately dirtied walls or internal complexity of the sample shorten the relaxation time. This is not to be expected from relaxation in the bulk of the gas due to either He^3 or impurities, but is to be expected from wall relaxation. Although the justification and analysis cannot be presented here, the relaxation times are quantitatively consistent with the assumption that there exists a monolayer of He^3 on the walls in which the relaxation occurs,[12] and that each atom has probability of sticking per collision of greater than about 1/2. The ef-

ficacy of various wall coatings has not been completely determined but we have had good results with the treatment described, and we have found that bare Pyrex is certainly inferior. The sample described is obviously a superior one and exhibits not only longer nuclear relaxation time than others but also shows much better optical pumping signals than usual.

This technique should be applicable to any noble gas having a nuclear moment. It should not be applicable to any molecular gas since the nuclear relaxation time is greatly shortened by the interaction between the nuclear moments and the rotating charge distribution of the molecule. Although the amount of polarization (0.01%) is as yet not useful in nuclear physics experiments, we hope to improve it by better wall coatings, by the use of hotter walls to reduce the dwell time of the He^3, or possibly by the use of "mixing" frequencies[13] to increase the transition rate of the Rb-He flips.

It has recently been suggested[14] and discussed[15] that a possible study of the relaxation time in a monatomic gas might give information about the electric dipole moment of the nucleus through its interaction with the electric field produced during collisions. It is tempting to think that this method of study of the He^3 relaxation would be useful in this respect, but the actual frequency and density conditions of this experiment are still several orders of magnitude away from the optimum conditions for study of this question, in spite of the fact that our observed relaxation times are longer than a shortest estimate[15] for such a relaxation mechanism. The wall relaxation again appears to be a limiting factor.

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†Now at the Laboratoire de Physique, Ecole Normale Supérieure, Paris, France.

‡Alfred P. Sloan Foundation Fellow.

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MODIFICATION OF EFFECTIVE-RANGE THEORY IN THE PRESENCE OF A LONG-RANGE POTENTIAL*

Larry Spruch, Thomas F. O'Malley, and Leonard Rosenberg
Physics Department, Washington Square College, and Institute of Mathematical Sciences
New York University, New York, New York
(Received September 16, 1960)

The great success of effective-range theory (ERT) in the analysis and interpretation of low-energy nuclear two-body scattering data has stimulated its application to a wide variety of other fields, in some cases perhaps somewhat less critically than might be warranted. It should therefore be stressed that there are limitations to the applicability of ERT in its usual form. For long-range forces, for example, the very form of the expansion as a power series in k^2 is incorrect.

The simplest derivation[1] of ERT for zero angular momentum starts with the identity

$$k \cot\eta = -1/A + \tfrac{1}{2} r(0,k) k^2,$$

where

$$\tfrac{1}{2} r(0,k) \equiv \int_0^\infty (v v_0 - u u_0)\, dr.$$

$u(r)$ and $u_0(r)$ are the solutions of the Schrödinger equation with the potential $V(r)$ at energies E and

0, respectively, which satisfy the boundary conditions

$$u(0)=0, \quad u(r)\to \sin(kr+\eta)/\sin\eta \equiv v(r) \quad \text{for} \quad r\to\infty,$$

$$u_0(0)=0, \quad u_0(r)\to 1 - r/A \equiv v_0(r) \quad \text{for} \quad r\to\infty.$$

The original derivations were concerned primarily with strong, short-ranged (range $\approx R$) potentials. It was argued that (1) the limits of integration could effectively be replaced by 0 and R, since $u(r)\approx v(r)$ and $u_0(r)\approx v_0(r)$ for $r>R$, and (2) in the region 0 to R, for E small compared to some average of $V(r)$ in that region, the replacement of $u(r)$ by $u_0(r)$ and of $v(r)$ by $v_0(r)$ introduces an error of order k^2.

One then obtains the shape-independent approximation,

$$k\cot\eta = -1/A + \tfrac{1}{2} r_0 k^2 + O(k^4),$$

where the energy-independent effective range, r_0, is defined by

$$\tfrac{1}{2} r_0 = \int_0^\infty (v_0^2 - u_0^2)\, dr.$$

This form of expansion cannot however be correct, for example, for a potential which behaves asymptotically as $(2\mu/\hbar^2)V(r)\sim -\beta^2/r^4$. (Effective potentials of this form arise, in the adiabatic approximation, in the scattering of charged particles by a neutral polarizable system.) For this potential, one finds that for $r\to\infty$

$$u_0(r)\sim \frac{r}{\beta}\sin\frac{\beta}{r} - \frac{r}{A}\cos\frac{\beta}{r}$$

$$\equiv v_{0p}(r) = \left(-\frac{r}{A}+1\right) + \frac{\beta^2}{2Ar} - \frac{\beta^2}{6r^2} + \cdots,$$

from which it follows that $r_0=\infty$. The approximations noted above are then clearly invalid for this potential. The point is that if $V(r)$ approaches zero faster than any power of $1/r$ the same will be true for $u_0(r) - v_0(r)$, whereas for the present potential $u_0(r) - v_0(r)$ approaches zero so slowly that the contribution to $r(0,k)$ from large values of r is anything but negligible.

The solution of the differential equation with a potential proportional to $1/r^4$ can be expressed in terms of Mathieu functions.[2] This enables one to derive the modified shape-independent approximation

$$k\cot\eta = -\frac{1}{A} + \frac{\pi\beta^2}{3A^2}k + \frac{4\beta^2}{3A}k^2\ln\left(\frac{\beta k}{4}\right)$$

$$+\left[\tfrac{1}{2}r_{0p} + \frac{\pi\beta}{3} + \frac{20\beta^2}{9A} - \frac{8\beta^2}{3A}\Psi(\tfrac{3}{2}) - \frac{\pi\beta^3}{3A^2} - \frac{\pi^2\beta^4}{9A^2}\right]k^2 + \cdots, \quad (1)$$

where

$$\tfrac{1}{2}r_{0p} \equiv \int_0^\infty [v_{0p}^2(r) - u_0^2(r)]\, dr,$$

and

$$\Psi(\tfrac{3}{2}) = \Gamma'(\tfrac{3}{2})/\Gamma(\tfrac{3}{2}) = 0.0365.$$

Equation (1) is valid for k real and non-negative (and small). For k real and nonpositive, a different asymptotic expansion of the Mathieu function must be used, and one finds that k must be replaced by $-k$ on the right-hand side of Eq. (1). $k\cot\eta$ is therefore even in k for real k, as it must be.

To the extent that the adiabatic approximation is valid for the low-energy scattering of ions[3] and even of electrons by neutral atoms or molecules, as seems to be the case, Eq. (1) as appropriately modified to include symmetry effects is applicable to these situations. In this connection, we note the recent application[4] of ERT in its usual form to the singlet scattering of low-energy electrons by hydrogen atoms. The authors of that paper, however, expanded about the energy of binding of the H^- ion and it so happens, for reasons which we will go into at a later time, that this is a reasonably accurate approach. This explains the good agreement between their numerical result for the singlet scattering length and that deduced through the use of a rigorous minimum principle.[5] We also note that the effects of the term linear in k on continuous absorption due to free-free transitions in hydrogen are not negligible.[6]

Another application of Eq. (1) is to the scattering of neutrons by charged systems.[7] Due to the small polarizability of the neutron, the β^2/r^4 term can be treated as a perturbation. [No such approximation was necessary for the derivation of Eq. (1).] Doing so, Thaler found the term linear in k.

For $L>0$ and $V(r)\propto 1/r^4$, even the leading term of ERT is not of the correct form. Thus, it is $k^2\cot\eta$ which approaches a constant as $k\to 0$, rather than $k^{2L+1}\cot\eta$ as is the case[8] for short-range potentials.

One cannot expand $k^{2L+1}\cot\eta$ as a power series in k^2 for any long-range potential $[V(r)\propto 1/r^n]$ though the point at which new types of terms appear depends upon the potential. For Van der Waals scattering at $L=0$, for example, new terms will appear between k^2 and k^4. New terms will also appear, taking $L=0$ again, in the expansion of $C^2k\cot\eta$, where C^2 is the Coulomb

penetration factor, for the scattering of charged particles in the presence of another long-range potential. This would, for example, be the case for the scattering of protons by a nucleus, though the coefficients of these new terms might be quite small since they are proportional to the polarizability of the nucleus. New terms will also appear in nucleon-nucleon scattering due to the long-range magnetic interactions; for n-p or p-p forces it is clear from our previous discussion that there will also be additional (though generally very small) terms due to the nucleon polarizabilities.

A detailed report is in preparation. This report will also include some applications, with emphasis on electron scattering by hydrogen.

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SINGLE-VIRTUAL-BOSON-EXCHANGE INTERACTION IN HIGH-ENERGY COLLISIONS*

Freda Salzman and George Salzman
Department of Physics, University of Colorado, Boulder, Colorado
(Received September 12, 1960)

Recent experimental evidence points towards the importance of a long-range interaction, i.e., $\sim m_\pi^{-1}$, in high-energy collisions of π mesons with nucleons and of nucleons with nucleons.[1] A natural model to investigate is that in which the two incident particles collide by exchanging a single virtual π meson. In this note it is shown that if these graphs dominate in collisions in which the square of the virtual pion's four-momentum is very small, then a simple field-theoretical model of such collisions results.

Because the kinematics and field theory discussed in this note are not restricted only to processes involving π mesons and nucleons, we consider a general inelastic two-particle collision as shown in Fig. 1.[2] Particles A and A', with four-momenta p_i and p_i', exchange a boson B with four-momentum Δ_i, and two groups of particles emerge: C, with total four-momentum P, and C', with total four-momentum P'. A and A' have rest masses w and w',

$$p_i^2 = -w^2 \quad \text{and} \quad p_i'^2 = -w'^2,$$

and the "rest masses" W and W' of C and C' are defined by

$$P^2 = -W^2 \quad \text{and} \quad P'^2 = -W'^2.$$

The over-all barycentric coordinate system (U) is defined by

$$\vec{p}_{iU} + \vec{p}_{iU}' = 0,$$

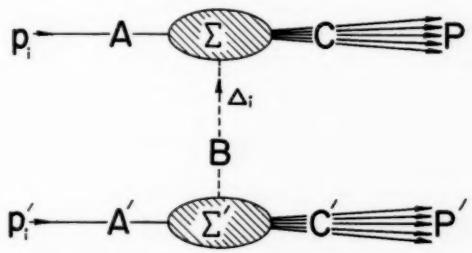

FIG. 1. A general binary inelastic collision in which particles A and A', with four-momenta p_i and p_i', interact by the exchange of a single boson B, with four-momentum Δ_i, leading to two groups of final state particles: C, with total four-momentum P, and C', with total four-momentum P'.

and the "scattering angle" θ in (U) is defined by

$$\vec{p}_{iU}\cdot\vec{P}_U = p_{iU}P_U\cos\theta.$$

From conservation of four-momentum at each of the two "vertices," Σ and Σ', one obtains

$$\Delta^2 \equiv \Delta_i^2 = [\Delta^2(U, w, w', W, W')]_{\min} + 4p_{iU}P_U\sin^2(\theta/2), \quad (1)$$

where U, the total energy in the system (U), is defined by

$$(p_i + p_i')^2 = -U^2.$$

This is but a slight generalization of the corresponding equation for elastic scattering, for which case $W=w$, $W'=w'$, and $(\Delta^2)_{\min}=0$. For the case that $w'=w$, e.g., proton-proton collisions, one obtains

$$(\Delta^2)_{\min}U^2 \approx (W^2-w^2)(W'^2-w^2), \quad (2)$$

provided that each of W^2, W'^2, w^2, and $(\Delta^2)_{\min}$ is much less than U^2. One finds:

(1) Δ^2 may be small in collisions in which a number of additional particles are produced. If, for example, A is a nucleon with laboratory energy 10^4 Bev and A' a nucleon at rest, so that $U\approx 140$ Bev, then $\Delta^2=m_\pi^2$ can occur with $W=W'\approx 4.5$ Bev for $\theta=0$.

(2) Δ^2 is small only for small "scattering angles." In the example just mentioned, as θ increases from 0° to 1°, Δ^2 increases from m_π^2 to about 80 m_π^2.

The main field-theoretical simplification results from the fact, which may at first appear surprising, that for small values of Δ^2 the virtual boson behaves kinematically as an incoming, almost real, particle, both in its "collision" with A as seen in the barycentric system (W) of the group of particles C, and in its "collision" with A' as seen in the barycentric system (W') of the group of particles C'.[2] It is incoming in the sense that it carries positive energy into the vertex. This may be seen by evaluating the two equations,

$$(P-\Delta_i)^2 = p_i^2 \quad \text{and} \quad (P'+\Delta_i)^2 = p_i'^2,$$

in the (W) and (W') coordinate systems, respectively. One obtains for the energy components of Δ_i in the (W) and (W') systems,

$$\omega_{\Delta W} = (W^2-w^2-\Delta^2)/(2W),$$

and

$$\omega_{\Delta W'} = -(W'^2-w'^2-\Delta^2)/(2W'). \quad (3)$$

It may be shown that if $W>w$ and $W'>w'$, then $\Delta^2>0$. Thus, for the case of interest, the boson B carries positive energy $\omega_{\Delta W}$ into the vertex Σ, as seen in (W), for $\Delta^2<W^2-w^2$, and carries negative energy $\omega_{\Delta W'}$ away from the vertex Σ', as seen in (W'), for $\Delta^2<W'^2-w'^2$. This result is possible because the four-vector Δ_i is spacelike and the sign of its energy component can change under the Lorentz transformation that connects the (W) and (W') coordinate systems.

In order to help visualize this result it is useful to consider a Minkowski diagram for energy E and momentum $\vec{p}$, for the case $\theta=0$, as is shown in Fig. 2, where z is taken to be the collision axis. Shown as dashed curves are the mass hyperboloids for real bosons, $\Delta^2=-m_B^2$; for real photons, $\Delta^2=0$; and for virtual bosons with $\Delta^2=m_B^2$. Shown as solid-line vectors are possible components of Δ_i in (W) and in (W'), and the components of the four-momentum of a real boson with almost the same energy and momentum, in (W), as Δ_i.[3] From Fig. 2 it is clear that the effect of the sequence of Lorentz transformations that carries (W) into (W') is quite different in the case of a virtual particle than in the case of a real particle.[4] The four-momentum of the real

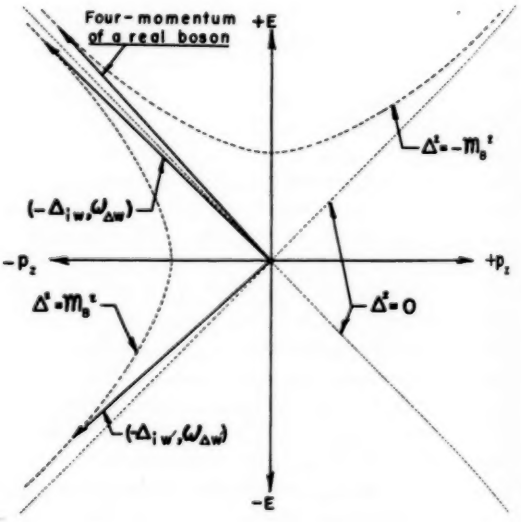

FIG. 2. Minkowski energy-momentum diagram illustrating the kinematical differences between real and virtual particles.

boson remains always on the hyperboloid $\Delta^2 = -m_B{}^2$, so that the sign of its energy component remains positive, while its three-momentum may change sign. For a virtual particle the sense of the three-momentum is invariant, but its energy may change sign.

If $\omega_{\Delta W}{}^2$ is large compared to $m_B{}^2$ and to Δ^2, then there is a real boson four-momentum with energy and three-momentum components in (W) very close to those of Δ_i, as is shown in Fig. 2. It is in this sense that the virtual boson behaves kinematically as an "almost real" particle at the Σ vertex in the system (W). Of course, the corresponding result is obtained at the Σ' vertex in the system (W'). The cross section for the process $A + A' \to C + C'$, as given by the graph of Fig. 1, will contain, according to the usual Feynman rules, the Lorentz-invariant vertex functions

$$\sum_f \bar{\sum}_i |\langle C|\Sigma|\Delta_i, p_i\rangle|^2,$$

and

$$\sum_{f'} \bar{\sum}_{i'} |\langle C'|\Sigma'|-\Delta_i, p_{i'}\rangle|^2,$$

where $\sum_f$ and $\bar{\sum}_i$ are the appropriate sums and averages over spins, and the Lorentz-invariant phase space factors $d\Omega_C$ and $d\Omega_{C'}$, where

$$d\Omega_C = (2\pi)^{-3(m+n)} \prod_{l=1}^{m} m_l \frac{d^3p_l}{E_l} \prod_{j=1}^{n} \frac{d^3k_j}{2\omega_j}$$

for the case in which C consists of m fermions and antifermions and n bosons, and similarly for $d\Omega_{C'}$. Because the virtual boson is an incoming, almost real particle in the system (W), it is reasonable to define

$$\sigma_{A+B\to C}(\Delta^2, W) \equiv \frac{(2\pi)^4 w}{2p_W W} \int d\Omega_C\, \delta^4(p_i + \Delta_i - P) \sum_f \bar{\sum}_i |\langle C|\Sigma|\Delta_i, p_i\rangle|^2, \tag{4}$$

where p_W is given by

$$W = (p_W{}^2 + w^2)^{1/2} + (p_W{}^2 + m_B{}^2)^{1/2}, \tag{5}$$

and similarly for $\sigma_{A'+\bar{B}\to C'}(\Delta^2, W')$. These are the "scattering" cross sections for the collisions of an almost real boson B, $(\bar{B})$ with the real particles A, (A'). This is the simplest definition consistent with the requirement that

$$\lim_{\Delta^2 \to -m_B{}^2} \sigma_{A+B\to C}(\Delta^2, W) = \sigma_{A+B\to C}(W),$$

the physical cross section. With these definitions one obtains for the differential cross section,

$$\frac{d\sigma_{A+A'\to C+C'}(\Delta^2, U)}{d(\Delta^2)} = \frac{2}{(2\pi)^3 p_{iU}{}^2 U^2} [D_{m_B{}^2}{}^C(\Delta^2)]^2 \int dW dW' p_W{}^{W^2} \sigma_{A+B\to C}(\Delta^2, W) p_{W'}{}^{,W'^2} \sigma_{A'+\bar{B}\to C'}(\Delta^2, W'), \tag{6}$$

where $D_{m_B{}^2}{}^C(\Delta^2)$ is the boson propagator which for the case of a π meson is $(\Delta^2 + m_\pi{}^2)^{-1}$, and the upper limits of integration depend upon Δ^2. This formula is expected to be valid for all processes in which large-impact-parameter collisions are dominant.

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[4]We are indebted to Professor P. Morrison for a stimulating discussion on this point.

RANGE OF PROTON-ANTIPROTON ANNIHILATION*

Maurice Lévy
Laboratoire de Physique Théorique et Hautes Energies, Orsay, Seine-et-Oise, France
(Received September 28, 1960)

Experimental data on elastic and inelastic scattering cross sections of antiprotons in hydrogen are now available up to energies of the order of several Bev.[1] Although the total p-$\bar{p}$ cross section is decreasing slowly when the energy increases, it still remains, at 2 Bev, of the order of twice the p-p cross section. Preliminary data[2] obtained at CERN tend to indicate that the limit of equality of the two cross sections which is predicted by the theorem of Pomeranchuk[3] is still not reached at 10.7 Bev. This can be taken as an indication that the pure annihilation process still remains fairly important at that high energy.[4]

Several theoretical investigations have been made on the p-$\bar{p}$ interaction, either phenomenologically[5] or on the basis of meson theory.[6-7] One of the most puzzling questions is the determination of the range of the annihilation region. Following familiar arguments of meson theory, this range is expected to be of the order of twice the nucleon Compton wavelength ($\sim 0.5\times10^{-13}$ cm, which is also the radius of the repulsive core in the nucleon-nucleon interaction) rather than of the order of the pion Compton wavelength. Since one cannot expect that the validity of a potential model or of the static meson field theory can be extended to the multi-Bev energy region we would like to present arguments based qualitatively on semiclassical concepts. In particular, a "maximum theorem" has been demonstrated[8] which relates the total and elastic cross sections to the maximum number of partial waves $(L+1)$ which contribute to the scattering process. The theorem can be expressed as follows:

$$(\sigma_{tot})^2/\sigma_{el} \leq 4\pi\lambda\!\!\!^{-2}(L+1)^2; \qquad (1)$$

it is obtained by making all the phase shifts purely imaginary:

$$\delta_l = i\beta_l, \quad (l \leq L) \qquad (2)$$

and varying the real parameter β_l so that $(\sigma_{tot})^2/\sigma_{el}$ is maximum. In other words, the maximum theorem is obtained when the interaction takes place in a purely absorbing region, the elastic contribution coming solely from shadow scattering; $L+1$ is therefore the number of partial waves which contribute to the inelastic process.

Now, if we consider all the experimental data between 200 Mev and 2 Bev, we find that, within 20% (which is certainly of the order of the combined experimental errors), the ratio $(\sigma_{tot})^2/\sigma_{el}$ remains practically constant and equal approximately to 300 mb. On the other hand, $(L+1)\lambda\!\!\!^{-}$ is roughly the range R of the interaction region.[9] Inequality (1) can then be understood as meaning that the interaction region remains approximately constant in the above energy range, and that $300 \text{ mb} \leq 4\pi R^2$, or

$$R \geq 1.5\times10^{-13} \text{ cm}. \qquad (3)$$

One could object to the above argument that, if there is an attractive real potential outside the annihilation region, particles of high angular momenta which strike the target well outside that region will be curved in and annihilated, so that the number given by Eq. (3) is more nearly equal to the range of the real scattering potential. This argument, however, does not hold if L is differently large, because the centrifugal barrier will prevent any such effect. The calculations of Ball and Chew,[6] for example, show that beyond $l=2$ the centrifugal barrier dominates the meson-theoretical scattering potential. Actually, if there is an attractive potential $V(r)$, classical arguments show that the minimum radius of interaction will be given by solving the equation:

$$R = \frac{(L+1)\lambda\!\!\!^{-}}{1+|V(R)|/E}. \qquad (4)$$

However, if there is an attractive real potential, the maximum theorem (1) will be modified, since the scattering phase shifts can no longer be purely imaginary. But this is merely a second order effect, as can be seen by the following calculation. We suppose that the scattering phase shifts contain a real part α which is independent of l:

$$\delta_l = i\beta_l + \alpha, \qquad (5)$$

and we vary β_l, keeping α fixed, so that $(\sigma_{tot})^2/\sigma_{el}$ is maximum. With the simplifying assumption on α, the partial wave contributions can again be summed, and we obtain the following

result:

$$\frac{(\sigma_{tot})^2}{\sigma_{el}} \leq \frac{4\pi\lambda\!\!\!^{-2}(L+1)^2\cos^2\alpha}{1+\frac{4\pi\lambda\!\!\!^{-2}(L+1)^2}{\sigma_{tot}}\sin^2\alpha\left[\frac{4\pi\lambda\!\!\!^{-2}(L+1)^2}{4\sigma_{tot}}-1\right]}, \quad (6)$$

which is a generalization of Eq. (1) to the case where phase shifts are not purely imaginary. In the limit of small α, this gives

$$(\sigma_{tot})^2/\sigma_{el} \leq 4\pi\lambda\!\!\!^{-2}(L+1)^2\left[1-\left(1-\frac{2}{t}\right)^2\alpha^2\right], \quad (7)$$

where we have put $t=\sigma_{tot}/\pi\lambda\!\!\!^{-2}(L+1)^2$. [Note that if one allows α to vary, the maximum of the right-hand side of Eq. (6) is obtained by putting $\sin^2\alpha=0$, which gives back Eq. (1).]

Now, if we assume that $V(r)$ is approximately constant for $r \geq R$ and if we replace the inequality of Eq. (1) by an approximate equality, we obtain, by combining Eqs. (1) and (4),

$$(\sigma_{tot})^2/\sigma_{el} \simeq 4\pi R^2\left(1+\frac{|V|}{E}\right),$$

valid to the first order in V. This linear dependence in $1/E$ agrees better with experiment than our previous assumption of a constant value of $(\sigma_{tot})^2/\sigma_{el}$. One finds that the experimental data can be fitted quite well by taking $R \simeq 1.43\times10^{-13}$ cm and $|V| \simeq 38$ Mev, which are rather reasonable values.

In order to understand the meaning of this result, one has to keep in mind that there can be two different contributions to the inelastic cross section: either the truly inelastic scattering with multiple production of pions, or the pure p-$\bar{p}$ annihilation. If the first process is dominant, then it is not surprising that the range which we have obtained should be of the order of the pion Compton wavelength. If, however, the main contribution comes from the second process, then we are facing again the puzzling result which we have indicated previously, namely that the radius of the annihilation region is of the order of $\hbar/m_\pi c$. There are, at present, some indications, which are however not at all conclusive, that it is the latter hypothesis which seems to be verified.

Preliminary data obtained at CERN on the total p-$\bar{p}$ cross section[2] give a value of 55 ± 2 mb at 10.7 Bev. If our arguments were to remain valid up to that energy, one should expect then an elastic scattering cross section of the order of 12 ± 4 mb. This appears reasonable, since one expects that the p-$\bar{p}$ and p-p elastic scattering cross sections should be approximately equal at high energy, because they both reduce to diffraction scattering. If one assumes also that the pure annihilation cross section at very high energy is roughly equal to the difference between the p-$\bar{p}$ and p-p total cross sections, one obtains for it about 15 mb. There remain 30 mb for the inelastic p-$\bar{p}$ scattering, so that the ratio of this process to pure annihilation should be approximately 2 to 1 at that energy.

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CONSISTENCY OF THE $K^+ \to \pi^+ + \pi^0 + \gamma$ RATE WITH THE $\Delta T = \frac{1}{2}$ RULE

N. Cabibbo and R. Gatto
Istituti di Fisica delle Università di Roma e di Cagliari, Laboratori Nazionali di Frascati del C. N. R. N., Frascati, Italy
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It was proposed by Gell-Mann and Pais[1] that in the decays of hyperons and K mesons into strongly interacting particles the isotopic spin change is given by $\Delta T = \frac{1}{2}$ when electromagnetic interactions are neglected. The subsequent detailed investigation of the implications of the rule[2] led to predictions which seem now to be satisfied by the most recent data.[3]

For $K^+ \to \pi^+ + \pi^0$ the $\Delta T = \frac{1}{2}$ rule predicts no rate at all in the absence of electromagnetic corrections. Experimentally the $K^+ \to \pi^+ + \pi^0$ rate is found to be almost two hundred times smaller than the rate for $K^0 \to 2\pi$. This result is in the direction of the prediction from the $\Delta T = \frac{1}{2}$ rule. The question then arises of discovering the mechanism which makes the amplitude for $K^+ \to \pi^+ + \pi^0$, arising from electromagnetic corrections of order e^2, large enough to account for the observed rate. Independently of this question it has been pointed out that the amplitude for $K^+ \to \pi^+ + \pi^0 + \gamma$ would be of the order e and might thus compete favorably with the amplitude for $K^+ \to \pi^+ + \pi^0$.[4] Such a possibility would strongly contradict present experimental evidence based on a few $K^+ \to \pi^+ + \pi^0 + \gamma$ events which can be accounted for in terms of the internal bremsstrahlung process accompanying the $K^+ \to \pi^+ + \pi^0$ decay.

In this note we show that there is no contradiction between the observed low frequency of $K^+ \to \pi^+ + \pi^0 + \gamma$ events and the $\Delta T = \frac{1}{2}$ rule. Specifically we show that the total $K^+ \to \pi^+ + \pi^0 + \gamma$ amplitude is not expected to be much larger than the internal bremsstrahlung contribution. This contribution would be of the order e^3, according to the exact $\Delta T = \frac{1}{2}$ rule, but it may thus compete favorably with the direct amplitude of order e. Such a rather unique situation would make very interesting a measurement of the π^+ spectrum of $K^+ \to \pi^+ + \pi^0 + \gamma$.

We shall construct an amplitude for $K^+ \to \pi^+ + \pi^0 + \gamma$ satisfying the requirements of gauge invariance. The total amplitude will consist of an internal bremsstrahlung amplitude (of order e^3 if $\Delta T = \frac{1}{2}$ holds exactly) and of a direct amplitude (of order e). The (necessarily off-shell) amplitude for $K^+ \to \pi^+ + \pi^0$ satisfying $\Delta T = \frac{1}{2}$ can be approximated by retaining only those terms with the lowest dependence on momentum transfer. Such an approximation should be valid if high-mass intermediate states contribute to the decay, as is expected. The form of the amplitude is then uniquely fixed by $\Delta T = \frac{1}{2}$ and corresponds to an effective Lagrangian,[5]

$$\mathfrak{L}' = \sqrt{2} f M \frac{\partial \Phi_+}{\partial x_\nu}\left[\varphi_+^* \left(\frac{\vec{\partial}}{\partial x_\nu} - \frac{\overleftarrow{\partial}}{\partial x_\nu}\right)\varphi_0\right] + \text{H.c.}, \quad (1)$$

where f is a constant, M is the K mass, Φ_+ is the K^+ field, and φ is the charged π field. The amplitude for $K^+ \to \pi^+ + \pi^0$ on the mass shell (which necessarily violates $\Delta T = \frac{1}{2}$) can be derived from the simplest Lagrangian,

$$\mathfrak{L}'' = g M^3 \Phi_+ \varphi_+^* \varphi_0 + \text{H.c.}, \quad (2)$$

on the basis that any structure effect from this vertex would certainly be negligible considering the smallness of the K^+ decay constant g. The current producing the radiative decay $K^+ \to \pi^+ + \pi^0 + \gamma$ includes contributions from the π and K currents and from the transition current derived from (1) through the substitution $\partial/\partial x_\nu \to \partial/\partial x_\nu - ieA_\nu$. The total amplitude is given by

$$\frac{e}{(4\pi)^2} \frac{M}{(M E_0 E_+ E_\gamma)^{1/2}} \epsilon_\mu j_\mu, \quad (3)$$

where

$$j_\mu = \left(\frac{p_\mu^{(+)}}{p_\nu^{(+)} \cdot q_\nu} - \frac{K_\mu}{K_\nu \cdot q_\nu}\right) \times [g M^2 - \sqrt{2} f q_\lambda \cdot (p_\lambda^{(+)} - p_\lambda^{(0)})]. \quad (4)$$

In (3) and (4), E_0, E_+, and E_γ are the energies of the final particles; ϵ_μ is the polarization vector; $p_\mu^{(0)}$, $p_\mu^{(+)}$, K_μ, and q_μ are the momenta of the final pions, of the K meson, and of the photon, respectively. It may be noted that the current (4) does not lead to any magnetic dipole emission; this is a consequence of our approximation which neglects higher structure terms. The pion spectrum from (3) and (4) is given by

$$\frac{1}{w_0}\frac{dw}{d\omega} = \left(\frac{e^2}{4\pi}\right)\frac{2qp}{\pi E \beta}[\mathfrak{B} + x\mathfrak{g} + x^2 \mathfrak{D}], \quad (5)$$

with

$$\mathfrak{B}=\frac{2}{q^2}\left[\frac{\omega}{p}\ln\frac{\omega+p}{\mu}-1\right],$$

$$\mathfrak{g}=\frac{\sqrt{2}}{M^2}\,\frac{E}{q}\left[6-\frac{\mu^2+2M\omega}{Mp}\ln\frac{M(\omega+p)-\mu^2}{M(\omega-p)-\mu^2}-\frac{M-4\omega}{p}\ln\frac{M-\omega+p}{M-\omega-p}\right],$$

$$\mathfrak{D}=\frac{E^2}{M^4}\left[2\,\frac{\mu^2+M\omega}{Mp}\ln\frac{M(\omega+p)-\mu^2}{M(\omega-p)-\mu^2}+4\,\frac{M-2\omega}{p}\ln\frac{M-\omega+p}{M-\omega-p}-12\right],$$

in the following notation: q is the photon momentum in the (γ,π^0) center-of-mass system, ω and p are the energy and momentum of π^+ in the K rest system, E is the total energy of (γ,π^0) in their center-of-mass system, β is the velocity of π^+ in the nonradiative decay, $x=f/g$, μ is the π mass, and w_0 is the rate for nonradiative decay. The expressions for E, q, and β are

$$E^2=M^2+\mu^2-2M\omega,$$

$$2Eq=M^2-2\omega M,$$

$$M^2\beta^2=M^2-4\mu^2.$$

In (5), $\mathfrak{B}$ is the bremsstrahlung term arising from the small amplitude (2) for $K^+\to\pi^++\pi^0$, $\mathfrak{D}$ arises entirely from the off-mass-shell amplitude (1), and $\mathfrak{g}$ is an interference term. The bremsstrahlung term coincides with that given by Good,[6] but the form of the other terms differs from estimates contained in reference 6 on the basis of dimensional and invariance considerations. In Fig. 1 we report $A\mathfrak{B}$, $Ax\mathfrak{g}$, and $Ax^2\mathfrak{D}$ with $A=qp/E\beta$, as given by (5), and for $x=15$. In the ratio x, the quantity g can be derived from the $K^+\to\pi^++\pi^0$ rate. A simplest model that gives a unique value for f is the following. The $K\to 2\pi$ decays are supposed to proceed through virtual dissociation of the K meson into a nucleon and an antilambda (for both K^+ and K^0); the Λ-decay interaction (subject to $\Delta T=\frac{1}{2}$) contains the derivative of the pion field. It then follows that $K^0\to 2\pi$ comes from an effective Lagrangian

$$fM\frac{\partial\Phi_0}{\partial x_\nu}\left(\vec{\varphi}\cdot\frac{\partial\vec{\varphi}}{\partial x_\nu}\right), \qquad (6)$$

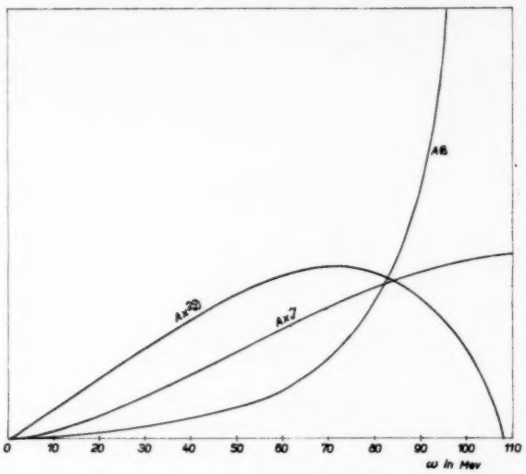

FIG. 1. The different contributions to the π^+ spectrum in $K\to\pi+\pi^0+\gamma$. $A\mathfrak{B}$ is the bremsstrahlung term, $Ax^2\mathfrak{D}$ is the direct term, and $Ax\mathfrak{g}$ is the interference between the two. The value of the parameter x is here chosen to be 15.

with the same f as in (1). The value of f can then be obtained by comparing the expression for the total $K^0\to 2\pi$ rate derived from (6) with its experimental value. In this way one obtains $x=\pm 15$. According to the choice of the sign, one finds that the number of $K^+\to\pi^++\pi^0+\gamma$ events in the best explored region of 55-75 Mev is bigger by a factor 1.6 (for negative interference term) or 5 (for positive interference) than the number predicted on the basis of internal bremsstrahlung alone (i.e., 1 event for the 8653 decays examined, among which two anomalous decays were reported). We consider our conclusion to be in agreement with experiment also, in view of our ambiguous determination of f. For instance, if our model to determine f is extended to include Σ particles, then one can derive in perturbation theory, assuming special forms for the hyperon decay Lagrangians,[7] cutoff-independent relations between f and the K^0-decay constant of (6). Such relations do not give unique results, however, because of the possibility of varying some signs in the hyperon decay Lagrangians.[8]

[1]M. Gell-Mann and A. Pais, Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics (Pergamon Press, New York, 1955).

[2]R. Gatto, Nuovo cimento 3, 318 (1956); G. Wentzel, Phys. Rev. 101, 1214 (1956).

[3]See F. S. Crawford et al., Phys. Rev. Letters 2, 266 (1959).

[4]M. Gell-Mann, Nuovo cimento 5, 758 (1957).

[5]We report briefly the argument. The rule $\Delta T=\frac{1}{2}$ implies that the final pions appear in the combinations

$$\epsilon_{ikl}\varphi^{(k)}\varphi^{(l)}, \quad \epsilon_{ikl}\frac{\partial\varphi^{(k)}}{\partial x_\nu}\frac{\partial\varphi^{(l)}}{\partial x_\nu},$$

$$\epsilon_{ikl}\frac{\partial\varphi^{(k)}}{\partial x_\nu}\varphi^{(l)}, \text{ or } \epsilon_{ikl}\varphi^{(k)}\frac{\partial\varphi^{(l)}}{\partial x_\nu}.$$

Of such expressions only the last two differ from zero and they are equal apart from sign.

[6]J. D. Good, Phys. Rev. 113, 352 (1959).

[7]O. Hori, Nuclear Phys. 17, 227 (1960).

[8]Particular models including Σ particles are being investigated by Dr. Bassetti, whom we wish to thank for useful discussions.

ESTIMATE OF THE NEUTRAL π-MESON LIFETIME

R. F. Blackie, A. Engler,* and J. H. Mulvey
The Clarendon Laboratory, University of Oxford, Oxford, England
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The lifetime of the π^0 meson may in principle be determined from an analysis of K^+-meson decays at rest in nuclear emulsion. Events are selected in which the $K_{\pi 2}$ mode, $K^+\rightarrow\pi^++\pi^0$, is followed by the decay of the π^0 meson to give a direct electron pair,[1] $\pi^0\rightarrow e^++e^-+\gamma$. Both the velocity and the direction of flight of the π^0 mesons are determined by the kinematics of the K-meson decay and the presence of the direct pair allows the point of decay of the π^0 to be found.

Harris, Orear, and Taylor[2] first proposed this method of estimating the π^0 lifetime, and using 12 events found in Ilford $G5$ nuclear emulsion stacks were able to set an upper limit to the lifetime at $\sim 7\times10^{-16}$ second. In the present experiment Ilford $L4$ emulsions of smaller grain size (~ 0.3 μ rather than the 0.6 μ typical of $G5$) have been used in an attempt to increase the spatial resolution of the method. The preliminary results, based on measurement of 26 events, suggest a finite mean lifetime $\sim 3\times10^{-16}$ second.

Thirty examples of K^+-meson decay accompanied by a direct electron pair have been found in a total of 27 000 K^+-meson endings examined. Measurements of the ionization of the secondaries have shown 2 events to be τ' or $K_{\mu 3}$ decays; there remains a possible 12% contamination due to decay by the $K_{\mu 3}$ and K_{e3} modes but the effect on the result is small.

The measurements have been performed using a projection apparatus to form an image of the event on a screen at a magnification ~ 7500. Each grain in the tracks of the π^+ meson, the electron pair, and the K-meson ending was then traced. The length of track used for the meson and electrons was ~ 30 μ. From the drawing, the coordinates of the center points of each grain were taken and this information was used to compute best-fit lines to the π^+ and electron tracks by minimizing $\sum h_i^2$, where h_i is the perpendicular distance from a grain center to the fitted line. This calculation was performed on the Oxford University Ferranti "Mercury" digital computer. At the same time the program computed the distance d between A, the foot of the perpendicular from the center of the last K-track grain to the π^+ line (the estimated origin of the π^0), and the point B, the intersection of the π^+ and one of the electron lines (the estimated point of decay); this calculation is illustrated by Fig. 1.

If the π^+-meson track has a dip angle α, the π^0-decay path is then $x=d\sec\alpha$. In those cases where both electron tracks of the pair were used, the estimate of d was obtained by taking an average using $1/\csc^2\theta$ as a weighting factor. Three drawings were made for each event and an average value used for the result.

This method of determining the mean decay

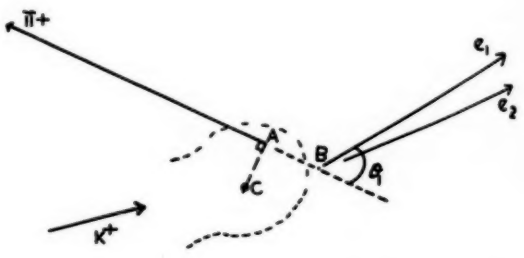

FIG. 1. Diagram showing the procedure for estimating the projected flight path of the π^0 meson. C is the center of the last grain in the K-meson track which is indicated by an irregular broken line.

path is at all stages free from subjective bias.

Following a suggestion due to Harris, Orear, and Taylor,[2] the error, σ_i, in a measurement x_i was assumed to be related to the geometry of the ith event in the following way:

$$\sigma_i = \sec\alpha(a^2 + b^2 \csc^2\theta)^{1/2}.$$

$b \csc\theta$ is a contribution to the error from the determination of the π, e intersection B, and a is a term related to the error in A.

The constants a and b have been obtained from measurements on 30 cases of τ decay, using exactly the same procedure to determine a distance d for a pair of π tracks and the K ending. The values obtained were $a = 0.063$ μ and $b = 0.084$ μ.

The results of measurements on 26 of the Dalitz pair events have been analyzed. Using as a weighting factor the quantity $1/\sigma_i^2$, the weighted mean distance is 0.147 ± 0.042 μ. This corresponds to a mean lifetime of $(3.2 \pm 1) \times 10^{-16}$ second.

Since the distribution in true π^0 path length is exponential, the distribution in x_i is not normal and the weighted mean is not the most efficient estimator for the mean decay path. The likelihood function for the observed set of x_i and σ_i has therefore been calculated and the maximum likelihood estimate for the mean lifetime is 3.3×10^{-16} second. The curve in Fig. 2 shows the relative probability of obtaining this set of measurements as a function of the lifetime. The values fall to 1% relative probability at $\sim 2 \times 10^{-17}$ second and $\sim 10^{-15}$ second.

A recent estimate of the π^0 lifetime by a dispersion theory method[3] suggests that the present results of experiments on proton Compton scattering are consistent with a lifetime within the limits 5×10^{-19} second and 10^{-16} second.

A further study of the estimation of errors, and of confidence limits for the lifetime is in progress and a fuller account of the experiment will be published elsewhere.

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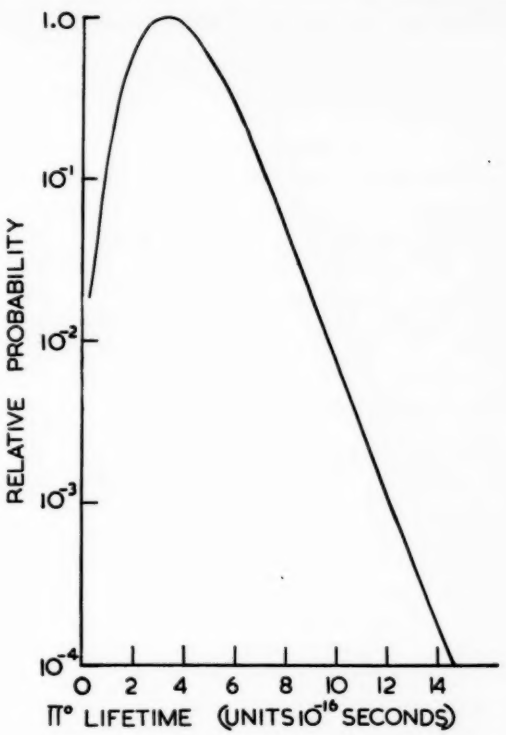

FIG. 2. The relative probability of observing the set of 26 measurements of x as a function of the π^0-meson lifetime.

*Now at the Physics Department, Duke University, Durham, North Carolina.

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[2]G. Harris, J. Orear, and S. Taylor, Phys. Rev. 106, 327 (1957).

[3]M. Jacob and J. Mathews, Phys. Rev. 117, 854 (1960).

π-π SCATTERING, NUCLEON STRUCTURE, AND π-N SCATTERING

J. Bowcock, W. N. Cottingham, and D. Lurié
CERN, Geneva, Switzerland
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The question of the effect of a π-π scattering resonance in the $J=1$, $T=1$ state on π-N scattering has recently received a good deal of attention. This problem has first been investigated by Frazer and Fulco[1] who derived the integral equations for the $\pi+\pi \rightarrow N+\bar{N}$ reaction amplitude involving the π-π phase shifts. These equations were applied by Frazer and Fulco to the problem of the isovector nucleon form factors.[2]

In a recent investigation[3] we have obtained single one-dimensional representations for the π-N scattering amplitudes by means of the Cini-Fubini method.[4,5] This representation includes all the results previously obtained by Frazer and Fulco and, furthermore, allows one to obtain definite predictions for the low-energy π-N phase shifts in which the effect of the $J=1$, $T=1$, π-π resonance is taken into account. We wish to report here on the comparison between the predictions of our theory and experiment and show that a good agreement between the two is obtained.

A conclusion opposite to ours has recently been reached by Frautschi[6] who finds a π-π effect much too large to fit the $P_{1/2}$ π-N scattering lengths. In view of this, we wish to discuss the point of view we have adopted in I and explain the reason for the discrepancy pointed out by Frautschi.

Let us write the π-π scattering amplitude in the $J=1$, $T=1$ state in a simple Breit-Wigner form:

$$f_{\pi\pi}=e^{i\delta_{\pi\pi}}\sin\delta_{\pi\pi}/q^3=\gamma/(t_R-t-i\gamma q^3), \qquad (1)$$

where $q=(\frac{1}{4}t-\mu^2)^{1/2}$ is the pion momentum in the center-of-mass system and t_R and γ are the constants describing the position and width of the resonance.

Consider now the π-N scattering channel and, for example, the p-wave isotopic-spin-flip amplitude. It has been shown in I that, in the limit of a narrow π-π resonance, the contribution to this amplitude arising from π-π resonance scattering is[7]

$$f_{11}^{(\pi\pi)}-f_{31}^{(\pi\pi)}=-\frac{9}{2t_R}\frac{M}{W}\left[\omega C_1F_1+k^2\left(C_2+\frac{C_1}{2M}\right)(F_0-F_2)\right], \qquad (2)$$

where W and k are the total energy and pion momentum in the c.m. system, $\omega=(k^2+\mu^2)^{1/2}$, μ and M are the pion and nucleon masses, and the functions F_α are given by

$$F_\alpha\left(\frac{2k^2}{t_R}\right)=\int_{-1}^{+1}\frac{y^\alpha dy}{1+2k^2(1-y)/t_R}.$$

This term has to be added to the contribution of the Born term and of the 33 resonance (which we shall call the "nucleon term").

The two constants C_1 and C_2 determine the size of the effect of the π-π resonance on π-N scattering.[8] The main cause of disagreement between the work in reference 6 and our own lies in the different value used for t_R and in the determination of the value of these two constants in the sense that the values used in reference 6 are much larger than ours. We wish, therefore, to discuss in some detail our procedure for determining these constants and, in particular, to clarify what information can be obtained from the experimental data on the electromagnetic structure of the nucleon.

The effect of the π-π resonance on the absorptive parts of the isovector nucleon form factors $G_1^V(t)$ and $G_2^V(t)$ is given, in the limit of a narrow resonance, by[9]

$$\mathrm{Im}G_1^V(t)=-\pi e(C_1t_R^{1/2}/\gamma)\delta(t_R-t),$$

$$\mathrm{Im}G_2^V(t)=-\pi e(C_2t_R^{1/2}/\gamma)\delta(t_R-t). \qquad (3)$$

It is clear that the comparison of Eq. (3) with the experimental data on the form factors, by means of either subtracted or unsubtracted dispersion relations, can only yield information on t_R and on the ratios C_1/γ and C_2/γ. The comparison was carried through in I by means of subtracted dispersion relations which lead to expressions for $G_1^V(t)$ and $G_2^V(t)$ of the Clementel-Villi[10] form:

$$G_1^V(t)=\frac{e}{2}\left(1+\frac{\eta_1 t}{t_R-t}\right),\quad G_2^V(t)=\frac{g_V e}{2M}\left(1+\frac{\eta_2 t}{t_R-t}\right), \qquad (4)$$

where $\eta_1 = -2C_1/t_R^{1/2}\gamma$ and $\eta_2 = -2MC_2/g_V t_R^{1/2}\gamma$. The expression (4) for $G_2^V(t)$ is known to give an excellent fit to the data with $\eta_2 = 1.2$ and the root mean square radius $r_0 = (6\eta_2/t_R)^{1/2} = 0.8\times10^{-13}$ cm.[11] Furthermore, the experimental near-equality of the charge and magnetic moment form factors gives $\eta_1 \simeq \eta_2$. These values yield[12]

$$t_R = 22.4\mu^2, \quad C_1/\gamma = -2.66\mu; \quad C_2/C_1 = 0.272\mu^{-1}.$$

It must be emphasized that as one has no independent determination of γ one cannot obtain from electromagnetic data alone the size of the effect of the π-π resonance on π-N scattering.

There is a theoretical possibility of determining C_1 and C_2 in terms of the π-N coupling constant f^2 and the 33-resonance parameters. This determination is based on the work of Frazer and Fulco on the $\pi+\pi \to N+\bar{N}$ reaction amplitude.[1] These authors have obtained the following expressions for the $\pi+\pi \to N+\bar{N}$ p-wave helicity amplitudes f_+ and f_-:

$$f_\pm(t) = \frac{F_\pi(t)}{\pi}\int_{-\infty}^{a} \frac{\mathrm{Im} f_\pm(t')dt'}{(t'-t)F_\pi(t')}, \tag{5}$$

where $\mathrm{Im}\, f_\pm(t')$ is known in terms of f^2 and the 33-resonance parameters and $F_\pi(t)$ is the pion form factor which is known in terms of Eq. (1). Equation (5) can in principle be used to determine C_1 and C_2. We wish to point out, however, that some of the integrals appearing in Eq. (5) are divergent and that even those which do converge depend strongly on high t' contributions where the expressions for $\mathrm{Im} f_\pm(t')$ can no longer be trusted. A determination of C_1 and C_2 along these lines has been carried out by Frautschi and Walecka who find $C_1 = -2.2$, $C_2 = -0.74$. This determination, together with the much smaller value of t_R ($\sim 11.5\,\mu^2$) used in reference 6 is the cause of the large disagreement with experiment pointed out by Frautschi. At the time paper I was being prepared, we had reached a similar conclusion as regards the s waves and we decided, for this reason, to abandon the determination of C_1 and C_2 by means of Eq. (5) which is, in our opinion, the weakest point in the theory. We therefore consider C_1 and C_2 as free parameters to be fitted to experimental data on π-N scattering and nucleon structure. From the discussion of the form factors it is seen that there is only one remaining free parameter, say C_1, which we shall determine by fitting the π-N s-wave phase shifts. The procedure, which we shall discuss below, yields[13]

$$C_1 = -1.0, \quad C_2 = -0.272\mu^{-1},$$

$$\gamma = 0.376\mu^{-1}, \quad t_R = 22.4\mu^2. \tag{6}$$

It is obvious that this choice of C_1 is indeed compatible with the data on the electromagnetic structure of the nucleon. Thus we feel that the criticism expressed in reference 6, that the agreement with the π-N phase shifts is obtained at the expense of failing to explain the nucleon structure, is not justified.

Let us finally discuss the comparison between our theory and the experimental phase shifts, restricting ourselves to the isotopic-spin-flip combinations which are only affected by the π-π interactions in the $T=1$ state. The expression for the s-wave amplitude is

$$f_s^{1/2} - f_s^{3/2} = -\frac{9}{2}\frac{M}{W}\frac{\omega}{2k^2} C_1 \ln\left(1+\frac{4k^2}{t_R}\right) + b\omega, \tag{7}$$

where b is an unknown parameter depending on the contributions from higher mass intermediate states. As shown in Fig. 1, the best fit is obtained for the values $C_1 = -1.0$ and $b = -0.05\mu^{-2}$. There is some departure from experiment in the medium energy region which might be due to a deficiency of the experimental data. It would be very important to have more experimental information in this energy region to clarify this point. The fact that the value $C_1 = 0$ is clearly excluded shows the necessity of a $J=T=1$ π-π interaction in order to explain the s-wave phase shifts.

The comparison between theory and experiment for the $f_{11} - f_{31}$ combination is shown in Fig. 2. The excellent agreement between theory and experiment is due to the presence of the pion term [Eq. (2)] in which there are, of course, no remaining free parameters. It should be noted that the nucleon term alone even has the wrong sign.

For the $J=3/2$ p-wave amplitudes, the pion term is considerably smaller than for the $J=1/2$ ones: no essential deviation from the usual effective-range formula is to be expected.

If one considers $f_{D_{3/2}}^{(1/2)} - f_{D_{3/2}}^{(3/2)}$ (Fig. 3), one finds that, as for p waves, the nucleon term by itself gives a contribution of the wrong sign and that a π-π interaction is definitely needed to fit experiment. Because of the 600-Mev resonance for the $D_{3/2}^{(1/2)}$ wave, the rescattering correction

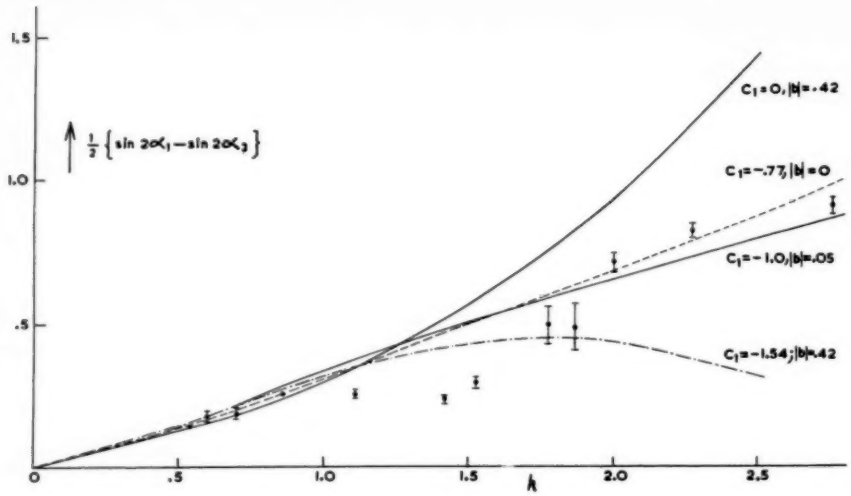

FIG. 1. The isotopic-spin-flip combination for s waves for several values of C_1 and b. The experimental points are from 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958); Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished); A. Stanghellini, Nuovo cimento 10, 398 (1958); W. D. Walker et al., Phys. Rev. 118, 1612 (1960).

is sizable and its inclusion improves the agreement between theory and experiment.

In conclusion, the introduction of a $J=T=1$ pion-pion resonance with the values of the parameters given in Eq. (6) gives a very satisfactory agreement with the existing experimental data on the isovector form factors and the π-N phase shifts. The remaining problem is to explain the discrepancy between our "experimental" values of C_1 and C_2 and the "theoretical" ones given in reference 6. We wish to point out that if the calculation in reference 6 is carried out using the value $t_R = 22.4\mu^2$ (suggested by a more careful analysis of the experimental data on the form factors[14]), the discrepancy is reduced to a factor ~ 2.[15] This discrepancy is due, in our opinion, to imperfect knowledge of the high-energy contribution to the integrals in Eq. (5), which can cause considerable errors in their evaluation.[16] It is hoped, therefore, that a more reliable method of solution of the Frazer-Fulco equations would lead to values of C_1 and C_2 closer to those proposed in this paper.

We wish to thank Professor S. Fubini for having read the manuscript and for many fruitful discussions.

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[2]W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960).

[3]J. Bowcock, W. N. Cottingham, and D. Lurié, Nuovo cimento 16, 918 (1960); hereafter called I. See

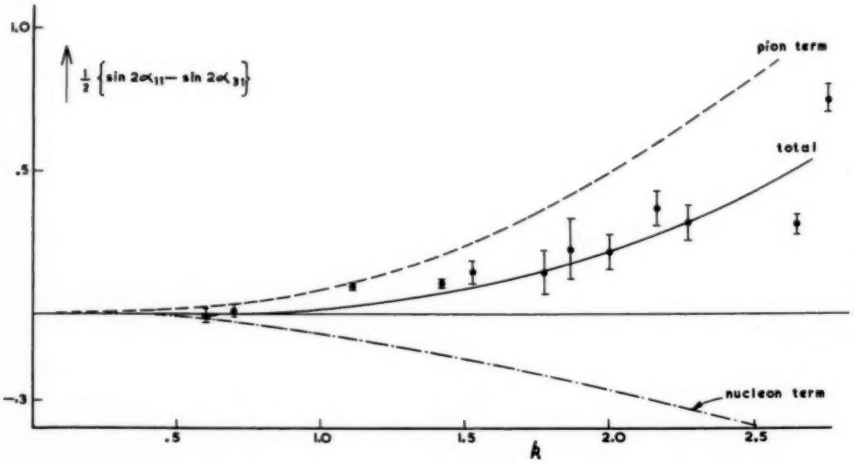

FIG. 2. The isotopic-spin-flip combination for the $P_{1/2}$ waves. The theoretical curve is drawn taking $C_1 = -1.0$. The experimental points are from the same data as Fig. 1.

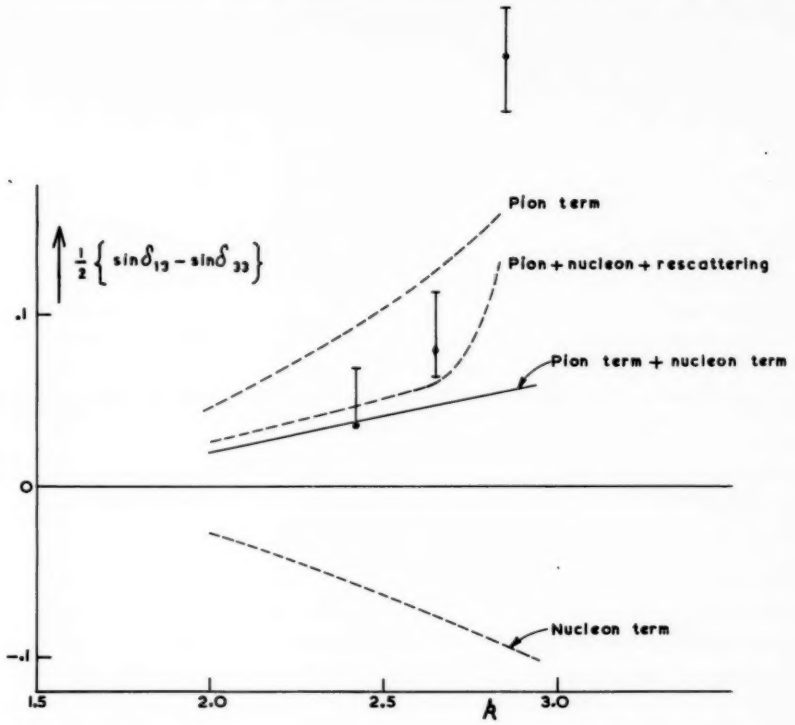

FIG. 3. The isotopic-spin-flip combination for the $D_{3/2}$ waves with $C_1 = -1.0$. The experimental points are from the same data as Fig. 1.

also the report by M. Cini presented at the Annual International Conference on High-Energy Physics, 1960 (unpublished).

[4]M. Cini and S. Fubini, Ann. Phys. 3, 352 (1960).

[5]Subsequent investigations on this question have been carried out by J. Hamilton and T. D. Spearman (to be published); S. C. Frautschi and J. D. Walecka (to be published); and by W. R. Frazer and J. R. Fulco (to be published).

[6]S. C. Frautschi, Phys. Rev. Letters 5, 159 (1960).

[7]As usual, f_L denotes $\exp(i\delta_L)\sin\delta_L/q$.

[8]C_1 and C_2 can be translated into the language of reference 6 by means of $C_1 = \gamma_1/\pi$, $C_2 = \gamma_2/M\pi$.

[9]We recall that $G_1^V(t)$ and $G_2^V(t)$ are normalized to $G_1^V(0) = e/2$, $G_2^V(0) = g_V e/2M$.

[10]E. Clementel and C. Villi, Nuovo cimento 4, 1207 (1956).

[11]R. Hofstadter, F. Bumiller, and M. R. Yearian, Revs. Modern Phys. 30, 482 (1958).

[12]The use of an unsubtracted dispersion relation for $G_2^V(t)$ as done in reference 2 would lead (for a small γ) to a Yukawa form factor which corresponds to setting $\eta_2 = 1.0$ in Eq. (4). The fit to the data is less satisfactory and requires $r_0 \simeq 1 \times 10^{-13}$ cm giving $t_R \simeq 12\mu^2$. This accounts for the difference in the values of t_R obtained in references 2 and 3. It may be pointed out that the error in t_R introduced by the narrow-resonance approximation is of the order of 10%.

[13]This value of C_1 is based on a more careful analysis of the experimental data and is somewhat larger than the value given in I.

[14]Evidence in favor of a $T = J = 1$ pion-pion resonance with $t_R \simeq 20\mu^2$ can be found in high-energy pion-nucleon collisions: I. Derado, Nuovo cimento 15, 853 (1960); F. Selleri, Nuovo cimento 16, 775 (1960); P. Carruthers and H. A. Bethe, Phys. Rev. Letters 4, 536 (1960); E. Pickup, F. Ayer, and E. O. Salant, Phys. Rev. Letters 5, 161 (1960); and from nucleon-antinucleon annihilation, see F. Cerulus, Nuovo cimento 14, 827 (1959).

[15]The evaluation of C_1 and C_2 in reference 6 is nearly insensitive to the value of t_R.

[16]As stated in a thesis by P. Cziffra, University of California Radiation Laboratory Report UCRL -9249 (unpublished), a similar conclusion was reached by D. Y. Wong. He compared the value of $f_+(0)$ calculated using Eq. (5) with the value as accurately known from a sum rule which gives $f_+(0)$ directly in terms of π-N total cross sections. He found that in order to get agreement between the two values, the "partial wave" estimation of $\mathrm{Im} f_+(t')$ had to be used for values of t' much larger than where this method of determination is valid. This confirms that the calculation of C_1 and C_2 using the unsubtracted relations (5) and the partial wave expansion is unreliable.

$\Delta I = \frac{1}{2}$ RULE

Abdus Salam
Imperial College, London, England

and

J. C. Ward
Carnegie Institute for Technology, Pittsburgh, Pennsylvania
(Received September 20, 1960)

One attractive way of formulating the $\Delta I = \frac{1}{2}$ rule is to postulate the existence of "spurions"[1]—neutral isospinor particles carrying no energy and momentum which are emitted or absorbed in decays of strange particles. We wish to point out that conventional field theory contains a perfectly natural realization of spurions. Consider the decay mode $K_1^0 \to \pi^+ + \pi^-$. If the Lagrangian for the process is $f_\omega K_1^0 \pi^+ \pi^-$, it is easy to see that the vacuum expectation value of K_1^0 would not be zero. In fact $\langle K_1^0 \rangle \propto f_\omega$. If we now consider a seemingly isotopic-spin conserving T product of field operators, like $\Lambda N^\dagger \vec{\tau} \cdot \vec{\pi} K$, it not only possesses matrix elements for strong interactions like $N + \pi \to \Lambda + K$, but also a matrix element for the decay $\Lambda \to N + \pi$ (arising from $\Lambda N^\dagger \vec{\tau} \cdot \vec{\pi} \langle K \rangle$) consistent with the $\Delta I = \frac{1}{2}$ rule: $\langle K_1^0 \rangle$ thus gives a realization for the spurion.

One can perhaps reverse the thinking above and reinterpret isotopic-spin nonconservation in the following manner. Let us postulate that all fields possessing the same quantum numbers as the vacuum (and in particular K_1^0 for which $CP = +1$) do have nonzero expectation values. These expectation values define the so-called coupling constants (in this instance the weak coupling constant f_ω). Admitting only seemingly isotopic-spin conserving products of field operators, we obtain both isotopic-spin conserving and isotopic-spin nonconserving matrix elements from the same expression. Apparent nonconservation of isotopic spin is another expression for the fact that K_1^0 had nonzero expectation value.

It is possible to interpret parity nonconservation in the same manner. If there exist[2] K' mesons of opposite (strong) parity to K mesons, $\langle K_2'^0 \rangle \neq 0$. Thus $\Lambda N^\dagger \vec{\tau} \cdot \vec{\pi} \gamma_5 K'$ gives a decay matrix element for $\Lambda \to N + \pi$ of opposite parity to one arising from $\Lambda N^\dagger \vec{\tau} \cdot \vec{\pi} K$, leading to parity nonconservation in the decay phenomenon.

Details of a theory of coupling constants and symmetry properties based on the above ideas will be published elsewhere.

We are grateful to Professor R. G. Sachs for the hospitality of the Summer Institute at the University of Wisconsin.

[1] G. Wentzel, Proceedings of the Sixth Annual Rochester Conference on High-Energy Nuclear Physics (Interscience Publishers, New York, 1956).

[2] M. Gell-Mann has re-emphasized the theoretical necessity of these particles for understanding weak interactions; Report at the Tenth Annual Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published).

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

INERTIAL MOMENT OF LARGE MANY-FERMION SYSTEMS WITH REPULSIVE INTERACTIONS. Ronald M. Rockmore, Brookhaven National Laboratory, Upton, New York (Received July 15, 1960).

The interaction effects on the moment of inertia of a large many-body fermion system moving under periodic boundary conditions have been explicitly derived in the second order of particle-particle coupling for the case of nonsingular repulsive interparticle forces. The derivation is facilitated by the use of a new Hamiltonian constructed by canonical transformation. This transformation is determined by requiring that it remove the compensating effects of pair excitation and that the new interaction operators of first and second order in the cranking field be also of first order in particle-particle coupling. A graphical analysis is presented in which those classes of diagrams which yield corrections vanishing relative to the rigid moment in the limit $L \to \infty$ are distinguished. Some remarks on the related problem of diamagnetism are made. The possible application of the method of canonical transformation to related many-body problems is indicated.

THEORY OF THE COMPRESSIBILITY OF SOLID He^4 AND He^3 AT 0°K. Newton Bernardes, Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa (Received August 8, 1960).

A quantum mechanical variational method is used to calculate several properties of a close-packed lattice phase of solid He^4 and He^3 at 0°K. The method consists in expressing the expectation value of the interatomic potential energy, with respect to a Heitler-London trial wave function, as a series of powers of the mean square deviation of an atom from its lattice site. Due to the small mass of a helium atom this mean square deviation is relatively large and the series converges slowly. Three sets of numerical results are obtained by truncating the series after the first, second, and third term, respectively. A comparison of these results with the experimental data shows that the final results, i.e., after minimizing $\langle H \rangle$ with respect to the variational parameter, converge much faster than the expectation value series itself. The results include values for cohesive energy, sound velocity, compressibility, Debye temperature, and Grüneisen constant. The calculations are repeated for a body-centered cubic lattice, and no indication of a crystallographic phase transition is found.

GYROMAGNETIC RATIOS OF MANGANESE ALLOYS. G. G. Scott, Research Laboratories, General Motors Corporation, Warren, Michigan (Received August 8, 1960).

The gyromagnetic ratios of two different ferromagnetic alloys of manganese were measured by Einstein-de Haas experiments. The Heusler alloy Cu_2MnAl gave a g' value of 1.993 ± 0.002. The alloy MnSb gave a g' value of 1.978 ± 0.002.

GENERAL SPIN WAVE DISPERSION RELATIONS. R. F. Soohoo, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received August 4, 1960).

The general spin wave dispersion relation obtained by a simultaneous solution of the equation of motion of the magnetization and Maxwell's equations is given. In contrast to previous calculations, the effects of conductivity, relaxation, exchange, and propagation are all properly taken into account. The resulting algebraic equation, being biquadratic in the square of the wave number, k^2, has four possible pairs of solutions. Some of these solutions correspond to growing plane waves while others represent attenuated ones in the direction of propagation. Whereas an analytical solution for k could be easily obtained for the special case where the wave vector $\vec{k}$ is in the direction of the static magnetic field ($\theta_k = 0$), the solution for the cases where $\theta_k \neq 0$ could be conveniently obtained only by numerical solution. The solutions for the latter cases have been obtained by using an IBM 709 computer and

some of the representative results are given in this paper in graphical form. When relaxation and eddy current damping are neglected, our result reduces to that of Herring and Kittel in the static limit ($\omega \to 0$). Furthermore, it was found that the uniform precessional mode ($k = 0$) can truly exist only under very special conditions, namely, under the condition of zero permeability for one of the two normal modes in a gyromagnetic medium.

ELECTRICAL STRUCTURE OF PbS FILMS. D. P. Snowden* and A. M. Portis, Department of Physics, University of California, Berkeley, California (Received April 18, 1960; revised manuscript received September 2, 1960).

The electrical properties of chemically deposited lead sulfide films have been investigated as a function of frequency from dc into the microwave range. Conductivity and Hall mobility measurements have been made on both the dark and the photoexcited carriers. The dark conductivity is independent of frequency below 100 Mc/sec. It increases by almost an order of magnitude over a decade in frequency and then becomes constant again at higher frequencies. This behavior is typical of a material whose low-frequency conductivity is limited by barriers. Surprisingly the photoconductivity behaves quite differently. The photoconductivity has a hump around 100 Mc/sec but otherwise is relatively independent of frequency. The Hall mobilities of the dark and photo-carriers are equal at dc in agreement with earlier studies. Microwave Hall measurements indicate that the dark carriers have about the same apparent mobility at 10 kMc/sec as at dc. It is shown that the conventional barrier model with a photoinduced change in carrier concentration cannot account for the behavior of these films. Nor can a model in which the barrier height is modified by illumination. The present studies suggest that there are connecting channels through the films and that the photoresponse of the films is determined by the behavior of these channels. A simple model is constructed and its parameters are adjusted to fit the conductivity data and the dc Hall mobility. As a check on the model the measured microwave Hall mobility is compared with the predicted value. Good agreement is found for high-quality commercial films. The hump observed in the photoconductivity follows naturally from this model, being associated with a modulation of the barrier capacitance. Studies of the photoconductive decay yield the same time constant at dc and microwave frequencies, supporting the picture that the same photocarriers are responsible for the dc and microwave photoconductivity.

*Now at John J. Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics, San Diego, California.

MOMENTUM DISTRIBUTIONS OF PHOTONS FROM POSITRONS ANNIHILATING IN ALKALI HALIDES. A. T. Stewart* and N. K. Pope, Chalk River Laboratories, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada (Received August 2, 1960).

The angular correlation of photons from the two-gamma decay of positrons in all sodium halides and all alkali chlorides has been measured. The data yield the momentum distribution of the two gamma rays which is also the momentum distribution of the annihilating electron-positron pairs. It is seen that the positive ion has very little influence on the momentum distribution in comparison with the negative ion. These distributions in momentum are compared with the distributions calculated from various models of electron and positron wave functions in the crystal. For fluorides and chlorides the electron-positron wave-function products which yield fits to the experimental data resemble Hartree-Fock free-ion electron wave functions. Both the wave-function products and the free-ion wave functions yield density distributions which are similar to the electron densities obtained by x-ray measurements. This conclusion makes improbable a model of single-particle wave functions describing both the electrons and the positron tightly bound to the negative ion.

*Now at Dalhousie University, Halifax, Nova Scotia.

SCATTERING OF CONDUCTION ELECTRONS BY LATTICE VIBRATIONS IN SILICON. Donald Long, Honeywell Research Center, Hopkins, Minnesota (Received August 4, 1960).

A theoretical model which assumes intervalley lattice scattering by phonons of 630° and 190°K characteristic temperatures in addition to the usual intravalley acoustic lattice scattering has

been applied to the results of measurements of electrical conductivity, Hall effect, and weak-field magnetoresistance in the 30° to 350°K temperature range in samples of nearly pure n-type silicon. The model gives a good quantitative description of the results when the ratios of the coupling constants for the 630° and 190° phonons to the coupling constant for acoustic scattering perpendicular to an energy-spheroid axis are, respectively, about 2.0 and 0.15. The coupling constant for acoustic scattering parallel to a spheroid axis was found in an earlier study to be about 1.5 times that for the perpendicular direction. The magnitude of the acoustic contribution to the total lattice scattering mobility, as determined empirically here, is in approximate agreement with the predictions of deformation-potential theory.

DEBYE TEMPERATURE OF ORTHORHOMBIC CRYSTALS AT 0°K. S. K. Joshi, Physics Department, The University, Allahabad, U. P., India (Received July 26, 1960; revised manuscript received September 20, 1960).

Explicit formulas are obtained for the Debye characteristic temperature of orthorhombic crystals by an extension of Houston's method of expanding the integrand involved in terms of appropriate harmonic polynomials. Θ(elastic) values have been computed for twelve crystals and the results are discussed.

SOME OPTICAL PROPERTIES OF CdSe SINGLE CRYSTALS. S. P. Keller and G. D. Pettit, International Business Machines Research Laboratory, Poughkeepsie, New York (Received August 15, 1960).

Optical transmission and reflection measurements were made on a single crystal of CdSe (crystal class C_{6v}) with light polarized parallel and perpendicular to the c axis. The absorption coefficients, $\mu_\perp$ and $\mu_\parallel$, were determined and are presented as functions of incident photon energy, at room temperature. The selection rule, $\mu_\perp > \mu_\parallel$, expected for all solids in the crystal classes C_{nv} and C_n, where $n = 3$, 4, or 6, is very well obeyed in CdSe. The absorption coefficients are not simple functions of the photon energy, but are nearly exponential and almost follow Urbach's rule.

The fluorescence of the single crystal has been measured at 77°K. It is peaked at 920 mμ and it is slightly polarized perpendicular to the c axis. The results are similar to what has been obtained for hexagonal ZnS.

TEMPERATURE DEPENDENCE OF THE NUCLEAR QUADRUPOLE SPECTRUM OF Nb^{93} IN FERROELECTRIC $KNbO_3$. Robert R. Hewitt, Department of Physics, University of California, Riverside, California (Received June 27, 1960; revised manuscript received September 1, 1960).

The nuclear electric quadrupole interaction of Nb^{93} in ferroelectric $KNbO_3$ has been measured as a function of temperature in the tetragonal, orthorhombic, and rhombohedral phases. The values of $(1/\nu)(d\nu/dT)$ for these three phases are approximately -10^{-3}, -10^{-3}, and -10^{-4} per degree centigrade, respectively. In the tetragonal phase at 220°C, e^2qQ/h has been measured at 24 Mc/sec and the asymmetry parameter η is zero. An ionic calculation of field gradients in the tetragonal phase has been made including point-charge and induced-dipole contributions. The results of this calculation give the Sternheimer antishielding factor $(1-\gamma_\infty)$ as 15 in good agreement with its estimated value of +16. The temperature dependence of the electric field gradient at the Nb^{93} site as calculated from this ionic model is in agreement with the measured temperature dependence in the tetragonal phase.

MICROWAVE FARADAY EFFECT IN SILICON AND GERMANIUM. J. K. Furdyna, Northwestern University, Evanston, Illinois, and S. Broersma, University of Oklahoma, Norman, Oklahoma (Received August 1, 1960; revised manuscript received September 7, 1960).

The Faraday rotation and ellipticity in a system of quasifree carriers is discussed and applied to microwave measurements on semiconductors. The theoretical expressions for these effects are analyzed with a digital computer for various ranges of the magnetic field B, the mobility μ, the conductivity σ, the frequency ω, the collision time τ, and the dielectric constant of the host material. It is possible to simplify these expressions in certain limiting cases. For μB smaller than unity, the rotation and ellipticity are proportional to B. For μB larger than

both unity and $\omega\tau$, they decrease as B^{-1} and B^{-3}, respectively. A maximum occurs near $\mu B = 1$ when $\omega\tau$ is small.

Rotation measurements on n- and p-type single crystals of silicon at room temperature, with resistivities from 0.5 to 40 ohm-cm, utilizing 9.6- and 35-kMc/sec radiation, are compared with the theory. Results for n-type germanium at 78°K, with μB varied up to about 6, agree with the calculated low- and high-field behavior. Faraday ellipticity measurements on n-type germanium crystals at 78°K are in qualitative agreement with the theory. In the case of small losses, the sign of the ellipticity is determined by the sign of the quantity $(4\omega\tau - \sigma/\omega\epsilon_{st}')$.

BAND STRUCTURE AND ELECTRON TRANSPORT OF GaAs. H. Ehrenreich, General Electric Research Laboratory, Schenectady, New York (Received August 8, 1960).

Existing experimental data on GaAs are reviewed and analyzed to yield the band structure in the vicinity of the band edges as well as the parameters characterizing the bands. On the basis of presently existing experimental evidence, chiefly the behavior of the optical band gap in Ga(As, P) alloys and the deduced pressure shift and density-of-states effective mass, it is thought likely that the subsidiary conduction band minima lie along [100] directions. Analytical expressions including nonparabolic effects are given for the energy and density of states of the [000] conduction band and used to obtain a better value of the effective mass from optical reflectivity data. The experimentally observed structure in the Hall effect in n-type material at elevated temperatures is shown to result from excitation of carriers into the subsidiary conduction band. Changes of resistivity with pressure are explained on the basis of an increase of the [000] effective mass at low pressures and the transfer of carriers to the subsidiary minima at higher pressures. The scattering mechanisms which are important in connection with transport phenomena are shown to be polar lattice scattering and charged impurity scattering in the highest mobility samples. The transport calculations leading to the mobility and thermoelectric power as a function of temperature and impurity concentrations are performed using variational techniques, and shown to agree well with experiment. The apparently low mobility in the subsidiary minima is attributed at least in part to the large effective mass and relatively small anisotropy ratio. An estimate shows scattering between the two conduction bands probably to be unimportant.

FERROELECTRIC PROPERTIES OF $BaLi_{2x}Al_{2-2x}F_{4x}O_{4-4x}$. T. G. Dunne and N. R. Stemple, IBM Research Laboratory, Poughkeepsie, New York (Received August 10, 1960).

Single crystals of $BaLi_{2x}Al_{2-2x}F_{4x}O_{4-4x}$, where $x = 0.15$ to 0.30, have been found to be ferroelectric with a Curie temperature in the range 127-153°C and a room temperature spontaneous polarization along the hexagonal c axis of approximately 0.1 μcoul/cm^2. The structure of these mixed crystals is unrelated to that of any previously known ferroelectric, but appears to be very similar to that of $BaAl_2O_4$.

RADIO-FREQUENCY HALL EFFECT IN A SUPERCONDUCTOR. G. Dresselhaus and M. S. Dresselhaus, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received August 5, 1960).

The rf Hall field in a superconductor is calculated for the applied dc magnetic field and the rf electric field both parallel to the metal surface. A two-fluid model and specular reflection boundary conditions are used to solve the transport problem in the limits of the classical and the extreme anomalous skin effects. It is found in both limits that the rf Hall effect is nonzero. The magnitude of the Hall angle in superconducting tin is estimated.

MOMENTUM DISTRIBUTION OF AN INTERACTING ELECTRON GAS. E. Daniel and S. H. Vosko,* Department of Physics, University of California, La Jolla, California (Received August 10, 1960).

The momentum distribution of an interacting electron gas at zero temperature is studied in the high-density region. Numerical computations are made for the graphs corresponding to the excitation of pairs, which are known to be equivalent to the random phase approximation of Bohm and Pines. Exchange graphs are discussed. The numerical results, extrapolated to actual elec-

tronic densities in metals, show that a large discontinuity in the momentum distribution still remains at the Fermi level.

*Now at McMaster University, Hamilton, Ontario.

ISOTOPE EFFECT IN Nb_3Sn. G. E. Devlin and E. Corenzwit, Bell Telephone Laboratories, Murray Hill, New Jersey (Received August 11, 1960).

The isotope effect on the superconducting transition of Nb_3Sn has been measured. The value of -0.08 for the exponential factor is only 1/5 of that of the elemental superconductors which have been reported.

FAR-INFRARED STUDIES OF BISMUTH. W. S. Boyle and A. D. Brailsford,* Bell Telephone Laboratories, Murray Hill, New Jersey (Received August 1, 1960).

The zero-field and magneto-optic studies of bismuth reported here cover a range of frequencies where the plasma effects are large and rapidly changing. It is found that the data lend themselves to an interpretation in terms of classical magnetoplasma theory when anisotropy is taken into account. The contribution of the free carriers to the optical properties can be isolated from the interband transitions and hence the static value of the dielectric constant, ϵ_0, the number of free electrons per cc, n, and the effective-mass parameters for the electron band can be determined when used in conjunction with the results of cyclotron resonance studies.

*Now at Scientific Laboratory, Ford Motor Company, Dearborn, Michigan.

EFFECTIVE MASS IN GRAY TIN FROM KNIGHT SHIFT MEASUREMENTS. W. E. Blumberg and J. Eisinger, Bell Telephone Laboratories, Murray Hill, New Jersey (Received July 25, 1960).

The nuclear magnetic resonance of Sn^{119} in gray tin was measured between 200°K and 300°K and the effective g factor was found to increase by about 4 parts in 10^5 over this temperature range. This increase in Knight shift is thought to arise from an increase in the number of conduction electrons. By combining this result with Busch and Mooser's measurements of the magnetic susceptibility over the same temperature range, m_l, the single-valley effective mass of the lighter of the charge carriers, is found to be $(0.3 \pm 0.05)m$ if the electron g factor is 2. The experiment also allows one to determine the Knight shift without making a comparison with a diamagnetic substance. The Knight shift of intrinsic gray tin is found to be 6×10^{-5} at 300°K. The fact that the Knight shift is positive shows that the electron g factor is not negative as would be expected if the band structure of gray tin were similar to that of InSb.

NEUTRON DIFFRACTION STUDIES OF CHROMIUM-MODIFIED Mn_2Sb. W. H. Cloud and H. S. Jarrett, E. I. du Pont de Nemours and Company, Wilmington, Delaware, and A. E. Austin and E. Adelson, Battelle Memorial Institute, Columbus, Ohio (Received July 5, 1960).

Existence of a ferrimagnetic to antiferromagnetic phase transition with decreasing temperature in Mn_2Sb modified with 2.3 weight percent chromium is confirmed by neutron diffraction.

CONSIDERATION ON THE PROPAGATION AND GENERATION OF MAGNETOSTATIC WAVES AND SPIN WAVES. P. C. Fletcher and C. Kittel,* Hughes Research Laboratories, Malibu, California (Received August 3, 1960).

A discussion is given of the dispersion relation, magnetization distribution, and group velocity of magnetostatic waves in an infinite circular cylinder with the static magnetic field parallel to the cylinder axis. The dispersion relation of the modes with $e^{i\varphi}$ angular dependence is, for $kR \gg 1$,

$$\omega \cong \gamma H_0 + \gamma 2\pi M_s (x_i/kR)^2 + (D/\hbar)k^2,$$

where x_i is a root of $J_0(x) = 0$; R is the cylinder radius; and D is the exchange constant. The group velocity of magnetostatic pulses at low wave vectors is shown to be considerably higher than magnon velocities.

*Permanent address: Department of Physics, University of California, Berkeley, California.

MICROWAVE MODULATION OF LIGHT IN PARAMAGNETIC CRYSTALS. N. Bloembergen, P. S. Pershan, and L. R. Wilcox, Harvard University, Cambridge, Massachusetts (Received August 3, 1960).

The considerations of Dehmelt and several other workers about the modulation of light by radiofrequency signals in atomic vapors are extended to paramagnetic solids. It is shown that these materials, driven near a microwave resonance at low temperatures, may be used both to create and to detect modulation of light at microwave frequencies. Experimental design criteria are discussed at the hand of two numerical examples, modulated circular dichroism in ruby and modulated Faraday rotation in a broad class of ionic rare earth compounds. Some possible applications of microwave-modulated light are reviewed.

COLOR CENTERS IN CESIUM HALIDE SINGLE CRYSTALS. P. Avakian* and A. Smakula, Laboratory for Insulation Research, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received August 12, 1960).

Color centers have been investigated in the CsCl-type alkali halides. Cesium chloride, bromide, and iodide single crystals were grown from the melt and CsCl crystals also from solution. Coloration was produced by 130-kv x rays, 3.0-Mev electrons, and by electrolysis. In CsI coloration resulted from electrolysis only. The absorption of uncolored and colored crystals has been measured from 0.175 to 3.5 μ at 25°, -78°, and -190°C. After coloration all three crystals show one strong band in the visible (near infrared for CsI) and several weaker bands at shorter and longer wavelengths, which shift with temperature change. The spectral positions in mμ for a number of the bands at -190°C are:

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
CsCl	227		270	370	430	579	715	780	855	980
CsBr	241	~270	~315	390	480	646	(780)	840	(~930)	1055
CsI	270	330		425	535	750			1050	1185

The strongest band (λ_6) behaves similarly to the F band in the NaCl-type alkali halides. The half-width of the band (0.20 to 0.23 ev at -190°C) and its increase with temperature, the shifting of the band maximum toward longer wavelengths upon warming to room temperature (by ~0.08 ev), and the conversion by bleaching with light into other bands support the assignment of this band as the F band. The spectral position of the band maximum approximately follows the Mollwo relation ($\lambda_{max} = \text{const}\, d^n$, where d = interionic distance and $n = 2.5$).

Bleaching experiments suggest the assignment of the band λ_{10} as the M band and the bands between the F and M bands as R bands. The origin of the ultraviolet bands is still uncertain.

*Present address: Technische Hochschule, Stuttgart, Germany.

ONSET OF CORRELATION IN INITIALLY UNCORRELATED SYSTEM. I. R. Senitzky, U. S. Army Signal Research and Development Laboratory, Fort Monmouth, New Jersey (Received July 25, 1960).

The system under consideration is a number of molecules contained in a resonant cavity and isolated from external influences. The molecules are assumed to have two energy levels, and the molecular frequencies have a Gaussian distribution centered at the cavity frequency; the initial states of the molecules are uncorrelated. The onset of correlation in the molecular behavior is studied by examining the field in the cavity and the power emitted by the molecules for effects depending on the square of the number of molecules, in a perturbation theory approach.

It is shown that correlation effects manifest themselves in the fourth-order interaction. Both the correlation energy in the field and the correlation power emitted by the molecules approach steady-state values after transient periods determined by the relaxation time of the cavity and the frequency spread of the molecules. A physical picture of the correlation effects, as being due to induced emission produced by the lowest order spontaneous and thermal emission, is investigated and found to be approximately correct. The ratio of correlation energy to lowest order spontaneous emission energy is derived. An analysis is made of the dependence of the results on the initial states of the molecules, and interpreted in terms of the physical picture. The effect of the presence of a number of cavity modes, rather than a single mode, within the frequency spread of the molecules is investigated under simplifying assumption, and is shown to multiply the correlation effects by the square of the number of modes.

ZEEMAN EFFECT OF THE PURELY CUBIC FIELD FLUORESCENCE LINE OF MgO:Cr^{3+} CRYSTALS. S. Sugano,* A. L. Schawlow, and F. Varsanyi,† Bell Telephone Laboratories, Murray Hill, New Jersey (Received August 5, 1960).

Both transverse and longitudinal Zeeman effects are studied of the most conspicuous red emission line (14 319 cm^{-1}) of a MgO:Cr^{3+} single crystal. The Zeeman patterns are examined experimentally, with a magnetic field parallel to the [001], [110], and [111] axes, with linear polarizations parallel and perpendicular to each direction of the magnetic field and with circular polarizations around the magnetic field.

Comparing these patterns with those theoretically calculated under the reasonable assumptions, it is found that the line is a magnetic dipole radiation due to the $t_2^3\ ^2E - t_2^3\ ^4A_2$ transition of chromium ions with purely cubic environments. It is also found that the Jahn-Teller coupling of chromophoric electrons and lattice in the excited state may be ignored in explaining the Zeeman effect.

*On leave from the Department of Physics, Tokyo University, Tokyo, Japan.

†Present address: Department of Physics, The Johns Hopkins University, Baltimore, Maryland.

ION-ATOM POTENTIAL ENERGY FUNCTIONS OBTAINED FROM kev SCATTERING DATA. G. H. Lane* and E. Everhart, Physics Department, University of Connecticut, Storrs, Connecticut (Received August 15, 1960).

The potential energy functions between ions and atoms in collision are determined from experimental data. The ion-atom combinations studied include Ar^+ on Ar, Ne^+ on Ar, Ne^+ on Ne, He^+ on Ar, He^+ on Ne, and He^+ on He using differential cross-section measurements by Fuls et al. and Jones et al. made at 25, 50, and 100 kev. These data are analyzed using equations developed by Firsov. The potential energies are positive, are in the range of one- to sixty-thousand electron volts, and correspond to ion-atom separations of a small fraction of an angstrom.

The resultant potential energy curves are compared with a function derived by Firsov from a statistical model and they are found to fit fairly well. The fit to an exponentially screened Coulomb potential energy curve is not quite as good, but both these theoretical curves fall within the accuracy of the experimental data.

*Now at Norwich University, Northfield, Vermont.

ANTISHIELDING AND CONTRACTED WAVE FUNCTIONS. Gerald Burns, International Business Machines Research Laboratory, Poughkeepsie, New York, and E. G. Wikner, John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic, San Diego, California (Received August 5, 1960).

The antishielding factors, γ_∞, of several contracted negative ions are calculated and compared to the apparent experimental values. Although the use of contracted wave functions results in considerably smaller γ_∞'s, the agreement is still not encouraging for the negative ions. Interrelations are found among the wave functions contracted by different means. Also, γ_∞ of several closed-shell positive ions and two ^6S-state ions (Mn^{+2} and Fe^{+3}) are calculated so that information from measured quadrupole coupling constants can be obtained. These positive-ion results along with other published values should enable γ_∞ of all closed-shell ions to be estimated. The results are used to determine the nuclear quadrupole moment of Fe^{57m} from the published value of the quadrupole coupling constant in Fe_2O_3. This result is discussed.

THOMAS-FERMI TECHNIQUE FOR DETERMINING WAVE FUNCTIONS FOR ALKALI ATOMS WITH EXCITED VALENCE ELECTRONS. Harvey J. Brudner* and Sidney Borowitz, New York University, Department of Physics, Washington Square, New York, New York, and Institute of Mathematical Sciences, New York, New York (Received July 15, 1960).

A technique has been developed for the calculation of excited-state, one-electron wave functions based on the Thomas-Fermi statistical theory of the atom. This technique is applied to heavy alkali atoms for which Hartree-type solutions are complex and difficult to obtain. The Thomas-Fermi differential system for the alkali positive ions is accurately solved utilizing Milne's method, and the results are summarized. This Thomas-Fermi core potential is then used as a central field in the Schrödinger equation together with

the Heisenberg type of polarization energy correction. Angular dependence is assumed to be capable of separation, and two basic techniques for solving the radial Schrödinger equation are discussed, one due to Ridley, the other to Biermann and Lübeck. The general approach permits correction for penetration of the excited electron's orbital. The Biermann and Lübeck type solution also allows for the inclusion of a qualitative correction for exchange. The techniques are applied to the potassium atom with an excited valence electron in the 6*s* state. The results are encouraging when compared with a Biermann and Lübeck type calculation using a Hartree central potential done by Villars. The 7*s* state of the cesium atom which has not been obtained by Hartree central potential is also computed.

*Present address: College of Physicians and Surgeons, Columbia University, New York, New York.

INTERATOMIC REPULSIVE POTENTIALS AT VERY SMALL AND INTERMEDIATE SEPARATIONS. Adolf A. Abrahamson* and Robert D. Hatcher, New York University, New York, New York, and George H. Vineyard, Brookhaven National Laboratory, Upton, New York (Received August 10, 1960).

Using a minimal and a maximal principle, respectively, two approximate expressions for the interaction potential between atoms are given such that their mean, $\overline{U}(R)$, differs from the exact value, U_0, in the Thomas-Fermi-Dirac (TFD) approximation, by not more than 4% for the case of a two-center system; and by not more than 14%, in the Thomas-Fermi (TF) approximation, for the case of a three-center system. The respective limits of applicability of these potentials are discussed, and some of their applications are pointed out.

*Present address: Department of Physics, The City College of New York, New York, New York.

PARAMETERS OF SOME LOW-ENERGY NEUTRON RESONANCES IN PLATINUM. John R. Waters,* Rensselaer Polytechnic Institute, Troy, New York (Received August 5, 1960).

Measurements of the elastic scattering of neutrons from thin platinum foils were made as a function of neutron energy using a time-of-flight technique. Scattering areas were obtained for the 11.9-, 19.6-, 68-, and 96-ev resonances. These were combined with a thick-sample transmission measurement and transmission data from other workers to give values for the parameters of the resonances. Level spins and partial widths were derived by a least squares method.

*Now at the Physics Department, Princeton University, Princeton, New Jersey.

L/K-CAPTURE RATIO AND E_L/E_K FOR Ar^{37}. A. G. Santos-Ocampo and D. C. Conway, Purdue University, Lafayette, Indiana (Received June 27, 1960).

The L/K-capture ratio of Ar^{37} has been determined to be 0.103 ± 0.003 in a high-pressure, multiwire proportional counter. This value is in excellent agreement with the theoretical value of 0.100. The average energy of the L peak was found to be 273 ± 6 ev. When a new value of 265 ± 5 ev is obtained for the critical L absorption energy of Cl by interpolation and the effect of the Auger process is considered, it is concluded that the energy to produce an ion pair in Ar at 0.2 kev is 0.98 ± 0.04 times the value at 2.8 kev.

ENDOTHERMIC DEUTERON STRIPPING REACTIONS. I. STRIPPING NEAR THRESHOLD. E. K. Warburton, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey, and L. F. Chase, Jr., Lockheed Missiles and Space Research Laboratory, Palo Alto, California (Received August 15, 1960).

Wilkinson's explanation of the good agreement between theory and experiment for low-energy, low Q-value, (d,p) and (d,n) stripping reactions is restated from the point of view of the dispersion relations derivation of the Butler stripping formula. From this point of view the kinematical conditions which should lead to the least distortion of the stripping pattern appear in a simple way. As a consequence of Wilkinson's arguments it is proposed that the stripping mechanism should contribute significantly to endothermic (d,p) or (d,n) reactions near threshold. Some (d,n) threshold work is discussed and it is concluded that this work gives some evidence that the stripping mechanism is important near threshold. The information on the reaction mechanism

near threshold which can be obtained from γ-ray angular distribution measurements in $(d, p\gamma)$ or $(d, n\gamma)$ reactions is discussed and a general form for the angular distribution function for either plane wave or distorted wave stripping is developed. The $B^{11}(d, n)C^{12}$(15.1-Mev level) reaction is considered in some detail and it is concluded that the experimental evidence is consistent with the reaction proceeding via the stripping mechanism near threshold.

ENDOTHERMIC DEUTERON STRIPPING REACTIONS. II. $C^{12}(d, p_3\gamma)C^{13*}$ REACTION. L. F. Chase, Jr., R. G. Johnson, and E. K. Warburton,* Lockheed Missiles and Space Research Laboratory, Palo Alto, California (Received August 15, 1960).

The total cross section of the $C^{12}(d, p_3)C^{13}$ reaction ($Q = -1.13$ Mev) to the third excited state of C^{13} at 3.85 Mev was measured for deuteron energies between 1.7 and 3.1 Mev. The cross section was obtained from the yield of the 170-kev γ ray corresponding to the electric-dipole transition between the $5/2^+$, C^{13} 3.85-Mev level and the $3/2^-$, C^{13} 3.68-Mev level. The angular distribution of the 170-kev γ ray relative to the deuteron beam was investigated for the same deuteron energy region by measuring yields at 0° at 90° to the beam. The measured anisotropy is compared to the predictions of plane-wave and distorted-wave stripping. The γ-ray anisotropy for deuteron energies between 1.7 and 2.4 Mev is in good agreement with the predictions of plane-wave stripping. Seven compound nucleus resonances corresponding to known N^{14} energy levels were observed between deuteron energies of 2.4 and 3.2 Mev. The Breit-Wigner single-level formula is used to extract resonance parameters for these levels.

*Summer visitor; permanent address: Palmer Physical Laboratory, Princeton University, Princeton, New Jersey.

EVAPORATION OF CHARGED PARTICLES FROM HIGHLY EXCITED COMPOUND NUCLEI. W. J. Knox,* A. R. Quinton, and C. E. Anderson, Yale University, New Haven, Connecticut (Received July 6, 1960).

Energy spectra and angular distributions of alpha particles and protons emitted in the bombardment of Ni targets with 160-Mev oxygen ions have been measured. The initial compound nucleus is characterized approximately by $Z = 36$, $A = 75$, total excitation energy = 125 Mev, and rms angular momentum = $50\hbar$. The energy distributions have the shapes of evaporation spectra although they are displaced by several Mev toward lower energies compared to calculated evaporation spectra. The angular distributions are approximately symmetric about 90° and sharply peaked in the forward and backward directions. At all energies there is some excess of particles in the forward direction and at high energies the distributions are predominantly forward peaked. Most of the particles are believed to be evaporated from compound nuclei with angular distributions determined by the high angular momenta of the emitting systems. The angular distributions can be fitted by a theory with a single adjustable parameter which is related to the variation of level density with angular momentum and the incoming and outgoing angular momenta. The total cross section for alpha-particle production is 2.6 ± 0.6 barns and for protons 4.0 ± 1.0 barns. These cross sections are larger than the estimated cross section for compound nucleus formation, indicating that several charged particles are evaporated per interaction. About half the collision cross section is believed to result in compound nucleus formation.

*Present address: Department of Physics, University of California, Davis, California.

COULOMB ENERGIES OF CLOSED-SHELL NUCLEI FROM SHELL-MODEL WAVE FUNCTIONS. N. V. V. J. Swamy and V. K. Kembhavi, Karnatak University, Dharwar, India, and D. G. Galgali, Institute of Science, Bombay, India (Received June 1, 1960; revised manuscript received August 11, 1960).

Using harmonic oscillator wave functions, the Slater integrals have been evaluated and expressed in the form of summation formulas. The Coulomb energies of seven closed-shell nuclei are estimated using these integrals. These estimates are compared with those based on a statistical model and a trapezoidal model. The influence of exchange energy on the Coulomb radius of a nucleus is shown to be sensitive to the model used. Although the $Z^{4/3}$ variation of exchange energy appears to be a suitable characterization, present

estimates require a larger multiplying constant in the usually accepted expression for the exchange energy. The direct and "net" energies, computed from the three models, show very good agreement.

(He^4, Be^7) REACTION IN MAGNESIUM, ALUMINUM, TITANIUM, COBALT, AND COPPER FROM THRESHOLD TO 42 Mev. Richard H. Lindsay, Department of Physics, Washington State University, Pullman, Washington (Received August 11, 1960).

Cross sections for the production of Be^7 in the He-ion bombardment of Mg, Al, Ti, Co, and Cu have been measured in the 30-42 Mev energy range. The excitation functions for these reactions are presented. A study of the bulk (0°-90°; 90°-180°) laboratory angular distributions by the catcher foil technique of the Be^7 nuclei emerging from 2.0- and 1.85-mg/cm^2 magnesium targets and an examination of the approximate range-energy curves for the Be^7 particles in aluminum and magnesium indicates that the reaction proceeds through a compound nucleus. The experimental excitation function for the Al^{27}(He^4, Be^7)Na^{24} reaction is compared with calculations based on the nuclear evaporation model. The cross sections for the production of Na^{24} and Be^7 in the He-ion bombardment of aluminum are contrasted and the difference between the yields leads to an excitation function for the (He^4, He^{3}He^4) reaction.

DEUTERON STRIPPING AND PICKUP REACTIONS IN OXYGEN-16. Eldon L. Keller,* Sarah Mellon Scaife Radiation Laboratory, University of Pittsburgh, Pittsburgh, Pennsylvania (Received August 15, 1960).

The reactions O$^{16}(d, p)$O^{17} and O$^{16}(d, t)$O^{15} were studied by bombarding thin nickel oxide foils with 15-Mev deuterons from the University of Pittsburgh cyclotron. The reaction particles were magnetically analyzed and detected either by nuclear emulsions or by a CsI(Tl) scintillator. Angular distributions and absolute cross sections were obtained for the first six states of O^{17} and for the ground state of O^{15}. Reduced widths having values $\theta^2 = 0.045$, 0.16, 0.0024, 0.0024, 0.0071, 0.047, and 0.012, respectively, were extracted from a comparison of the data with the predictions of Butler stripping theory. The most notable results of the (d, p) experiment indicate that (1) the $7/2^-$ state at 3.846 Mev does not appear to be a good $1f_{7/2}$ single-particle state, (2) the $2p_{3/2}$ single-particle component seems to be fragmented over more than two states, and (3) the $1/2^-$ state at 3.058 Mev contains a $2p_{1/2}$ single-particle component. The results of the (d, t) experiment suggest a dependence of the $1p$ single-particle reduced width on Q value.

*Now at the Radiation and Nucleonics Laboratory, Westinghouse Electric Corporation, Pittsburgh, Pennsylvania.

NUCLEAR STRUCTURE OF $_{50}$Sn^{120}. H. Ikegami, Institute for Nuclear Study, University of Tokyo, Tanashi-machi, Tokyo, Japan (Received July 22, 1960).

The radiations from the 5.8-day Sb^{120m} have been investigated in detail, using the sector-type double-focusing magnetic spectrometers and the scintillation counters. The following measurements were performed: coincident gamma-ray spectra, directional correlations of 1.18-Mev γ−1.03-Mev γ, 1.03-Mev γ−0.20-Mev γ, 1.18-Mev γ−0.20-Mev γ, 0.20-Mev γ−0.09-Mev γ, 0.20-Mev Ke^-−0.09-Mev γ, and 0.20-Mev γ−0.09 Mev Ke^-. The 1.18-Mev γ−1.03-Mev γ polarization direction correlation, the conversion electron spectra, and the mean lives of the 2.41- and 2.50-Mev levels have also been investigated. The decay scheme of the 5.8-day Sb^{120m} proposed by McGinnis, which gives the following assignments to the excited states in Sn^{120}: 1.18 Mev (2+), 2.21 Mev (4+), 2.41 Mev (6+), and 2.50 Mev (7-), has been confirmed uniquely. These levels have been interpreted as arising from the neutron configurations $(2d_{3/2})^2(1h_{11/2})^2$ and $(2d_{3/2})^3(1h_{11/2})^1$. The lifetimes of the 2.41-, 1.18-, and probably 2.21-Mev levels are shorter by a factor 3−4 than those expected for single-proton transitions and may be explained by a neutron transition, assuming a somewhat larger effective charge of neutron in spherical nuclei. The high forbiddenness of the 0.09-Mev $E1+M2$ transition is ascribable to the simultaneous j- and l-forbidden transition between the states involving $(2d_{3/2})^3(1h_{11/2})^1$ and $(2d_{3/2})^2(1h_{11/2})^2$ configurations.

No appreciable effects of the nuclear finite size or any nuclear structure dependence on the

internal conversion process have been detected even for the highly forbidden 0.09-Mev $E1+M2$ transition.

In the decay of the 16-min Sb^{120}, the 1.18-Mev gamma ray has been detected. However, other gamma rays expected from the second level multiplet which is a characteristic feature of the vibrational model, have not been found.

NEUTRON-PROTON POLARIZATION AND DIFFERENTIAL CROSS SECTION AT 128 Mev. Russell K. Hobbie and Douglas Miller, Cyclotron Laboratory, Harvard University, Cambridge, Massachusetts (Received August 1, 1960).

A 42% polarized neutron beam of 128 Mev average energy has been used to measure the n-p differential cross section and polarization. A range telescope detected recoil protons from a liquid hydrogen target placed in the beam. The following values were obtained:

$\theta_{c.m.}$	σ	P
169.7	11.30 ± 0.37	+0.002 ± 0.021
159.3	9.04 ± 0.30	-0.045 ± 0.021
149.0	7.27 ± 0.24	-0.061 ± 0.021
138.8	6.26 ± 0.21	-0.072 ± 0.017
128.5	5.18 ± 0.17	-0.016 ± 0.021
118.4	4.28 ± 0.14	+0.054 ± 0.021
108.2	3.45 ± 0.11	+0.052 ± 0.025
98.1	2.85 ± 0.10	+0.186 ± 0.028
88.1	2.63 ± 0.09	+0.275 ± 0.028
78.1	2.78 ± 0.11	+0.384 ± 0.028

The errors quoted do not include the 7% uncertainty in the normalization of the cross-section data or the 8% uncertainty in the normalization of the polarization data.

NUCLEAR ORIENTATION OF Tb^{160}. C. E. Johnson,* J. F. Schooley, and D. A. Shirley, Lawrence Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received August 11, 1960).

Nuclear orientation of Tb^{160} in a single crystal of neodymium ethylsulphate has been observed by the anisotropic intensity distribution of the γ rays. The alignment arises from the coupling of the nuclear magnetic and quadrupole moments with the crystal field. The spin of Tb^{160} is shown to be 3, and the spin of the 1360-kev level in Dy^{160} is 2. The nuclear moments of Tb^{160} are $|\mu| = 1.60 \pm 0.25$ nm and $Q = +1.9 \pm 0.5$ barns.

*Present address: Atomic Energy Research Establishment, Harwell, England.

REACTION $C^{12}(\alpha,d)N^{14}$. Bernard G. Harvey and Joseph Cerny, Lawrence Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received August 15, 1960).

The energy spectra of deuterons from the reaction $C^{12}(\alpha,d)N^{14}$ have been measured. The reaction was induced by 47.5-Mev helium ions. Angular distributions are given for the deuteron groups corresponding to formation of the ground state and 3.95-Mev level of N^{14}.

Formation of the 2.31-Mev 0+, $T=1$ level of N^{14} was not observed; the cross section was less than 3% of the cross section for formation of the 1+, $T=0$ ground state. It is suggested that this cannot be interpreted as strong evidence for the conservation of isotopic spin for two reasons. First, several well-known $T=0$ levels were not populated to an observable extent. Second, the reaction leading to the $T=1$ level would involve a transition from a 0+ level to a 0+ level, and such transitions in (α,d) or (d,α) reactions involve difficulties with angular momentum and parity conservation in addition to nonconservation of isotopic spin, which requires $\Delta T = 0$.

REACTIONS $N^{14}(d,n)O^{15}$ (g.s.) AND $N^{14}(d,n)O^{15*}$ (6.79 AND 6.86 Mev). A. J. Elwyn,* J. V. Kane, S. Ofer,† and R. Pixley, Brookhaven National Laboratory, Upton, New York (Received August 4, 1960).

The angular distribution of neutrons from the reaction $N^{14}(d,n)O^{15}$ (g.s.) has been measured at 8 bombarding energies between 1.35 and 2.8 Mev. The same measurements have been made on the neutron groups (unresolved) from the reactions $N^{14}(d,n)O^{15*}$ (6.79 Mev) and $N^{14}(d,n)O^{15*}$ (6.86 Mev) at bombarding energies of 2.55 and 2.7 Mev. Direct-interaction mechanisms are thought to be responsible for most of the observed yields although resonance-like effects are also seen. Neutron yields at forward angles show approximate Butler stripping behavior at all energies

in addition to other unexplained processes. The observed yields at large angles can be partially explained by the exchange stripping reaction for the high-Q reaction but cannot be explained on this basis for the low-Q reaction.

*Present address: Argonne National Laboratory, Lemont, Illinois.

†On leave from the Hebrew University, Jerusalem, Israel.

NUCLEAR ZEEMAN EFFECT IN Sn^{119}. S. S. Hanna, L. Meyer-Schützmeister, R. S. Preston, and D. H. Vincent, Argonne National Laboratory, Argonne, Illinois (Received August 5, 1960).

Ferromagnetic alloys of manganese and tin have been used to obtain large internal magnetic fields at the tin nuclei. The resultant Zeeman splittings of the two low levels of Sn^{119} have been measured by observing the hyperfine spectra in the resonant absorption of the 24-kev radiation of Sn^{119}. In Mn_4Sn an internal field of roughly 40 koe was found. In this field the splitting of the ground state was observed but that of the excited state was unresolved. In Mn_2Sn an internal field of about 190 koe was produced. In this field additional structure was obtained which was attributed to the splitting of the excited state. The measured splitting gives a value of (0.78 ± 0.06) nm for the magnetic moment of the excited state.

SLOW-NEUTRON RESONANCE SPECTROSCOPY. II. Ag, Au, Ta. J. S. Desjardins,* J. L. Rosen, W. W. Havens, Jr., and J. Rainwater, Department of Physics, Columbia University, New York, New York (Received August 3, 1960).

The results of time-of-flight measurements of Ag, Au, and Ta resonance parameters are presented. Neutron widths are given for 79 levels in Ag to 728 ev, 55 levels in Au to 940 ev, and 62 levels in Ta to 330 ev. Radiation widths for many of the stronger levels, and in some cases resonance J values, are obtained. The reduced neutron width distributions are in good agreement with Porter-Thomas distributions for Au and Ta. The distribution for Ag shows a large excess of small $(2g\Gamma_n^0)$ values which are interpreted as p-wave resonances and correspond to a p-wave strength function $S_1 = (1.7 \pm 0.8) \times 10^{-4}$. The s-wave strength functions are $(0.51 \pm 0.09) \times 10^{-4}$ for Ag, $(1.5 \pm 0.3) \times 10^{-4}$ for Au, and $(1.84 \pm 0.34) \times 10^{-4}$ for Ta. The observation of p-wave Ag levels is interpreted as due to a $p_{1/2}$ strength-function resonance in this region. The level spacing distributions for Au and Ta agree with the expected two-population Wigner spacing distribution function but not with random or single Wigner distributions. The weighted average Γ_γ values are 0.148 ev, 0.170 ev, and 0.060 ev for Ag, Au, and Ta, respectively.

*Present address: Department of Physics, University of Rhode Island, Kingston, Rhode Island.

NUCLEAR RESONANCE FLUORESCENCE IN Cu^{63}: LIFETIMES OF EXCITED STATES AND THE SLOWING DOWN OF RECOILS (3-100 ev) IN CONDENSED MEDIA. J. B. Cumming, A. Schwarzschild, A. W. Sunyar, and N. T. Porile, Brookhaven National Laboratory, Upton, New York (Received August 10, 1960).

Measurements of the lifetimes of the 669-kev and 963-kev states of Cu^{63} have been performed using the nuclear resonance fluorescence technique. Self-absorption experiments yield mean lives of $(2.94 \pm 0.24) \times 10^{-13}$ sec for the spin $\frac{1}{2}$ 669-kev level and $(7.2 \pm 1.8) \times 10^{-13}$ sec for the spin $\frac{5}{2}$ 963-kev level. The resonance yields from sources inside Cu metal and in water solution were measured. The reduction of the resonance yields due to slowing down of the recoils from the β^+ decay was calculated with the use of the model of Vineyard et al. for the solid and with an elastic collision model for the liquid environments. These calculations are in good agreement with the measured yields and result in a unique spin assignment to the 669-kev state. The effect of the scatterer temperature on the resonance yield was shown to be in agreement with the "effective temperature" predicted by the theory of Lamb.

SECOND-DIFFERENCE ANALYSIS OF BREMSSTRAHLUNG YIELD CURVES. K. N. Geller, University of Pennsylvania, Philadelphia, Pennsylvania (Received August 11, 1960).

The second-difference weighting function of the bremsstrahlung spectrum obtained from an empirical isochromat at 15.1 Mev and corrected for thick-target effects is shown to behave like a delta function of width approximately equal to the

energy interval in analysis. This behavior greatly simplifies cross-section analysis of bremsstrahlung-induced activation curves when resonance phenomena are present. Application of the method to preliminary yield data for the (γ,n) reaction in nitrogen and oxygen gives good agreement with known level structure.

NUCLEAR SPIN AND MOMENTS OF 14-hr Ga^{72}. W. J. Childs, L. S. Goodman,* and L. J. Kieffer,† Argonne National Laboratory, Argonne, Illinois (Received August 4, 1960).

The hyperfine structure of the 14-hr isotope Ga^{72} has been investigated by use of atomic-beam magnetic-resonance techniques. The nuclear spin is found to be 3, the magnetic dipole moment $\mu_I = -0.132196 \pm 0.000013$ nm (subject to correction for a possible hyperfine anomaly), and the electric quadrupole moment $Q = +0.718 \pm 0.007$ barn.

*Present address: University of Heidelberg, Heidelberg, Germany.

†Present address: Atomic Physics Section, National Bureau of Standards, Washington, D. C.

NUCLEAR $E1$ PEAK ENERGIES. J. H. Carver and D. C. Peaslee, Australian National University, Canberra, Australia (Received March 28, 1960; revised manuscript received August 29, 1960).

The peak energies E_g of electric dipole giant resonances in photonuclear reactions considerably exceed the shell-model spacings E_s in the same nuclei: The discrepancy $\delta = (E_g - E_s)/E_s$ is of order unity, although the giant resonances are supposed to arise mainly from transitions of nucleons between successive shells. The present note attempts to understand δ by noting that $|E_g - E_h| \ll E_g$ and considering sum rule expressions for the harmonic energy E_h. These sum rule expressions are well known to yield $E_h = E_d + E_x$, where E_x is special to the $E1$ operator and the charge exchange component of nuclear forces. Theoretical and empirical arguments are adduced that $(E_g - E_s)$ comprises mainly E_x, plus two secondary corrections which happen practically to cancel. Comparison with experiment gives a constant $E_x \approx 8$ Mev and thence a satisfactory account of δ. Previous discussions of δ appear to have neglected E_x and hence have failed to obtain even the right order of magnitude of δ for real nuclei. The use of a constant E_x also allows an improved fit to the curve of $\int \sigma dE$ as a function of A. It is pointed out that the value of E_x is comparable in significance with the average nuclear potential V: by virtue of the $E1$ excitation mode, E_x represents the difference of even-parity and odd-parity two-nucleon interactions, while V represents a sum of even and odd interactions. The effective mass for the model ground-state wave function is treated as a derived quantity and turns out to be $M^*/M \gtrsim 1$; this large value is attributed to a Thomas shift associated with finite nuclear boundaries.

NUCLEAR EXCITATION BY SCATTERING OF 40-Mev ELECTRONS. W. C. Barber, F. Berthold,* G. Fricke,† and F. E. Gudden,† High-Energy Physics Laboratory, Stanford University, Stanford, California (Received August 1, 1960).

The electron energy spectra resulting from the scattering of 40-Mev primary electrons were measured for the purpose of studying nuclear excitations. Targets of Li^6, Be, C, and Si were employed at scattering angles of 132° and 160°; Pb was studied at a single angle of 160°. In addition to the elastic peaks, all spectra show peaks corresponding to excitation of the target nucleus into well-defined energy states. Peaks corresponding to known levels in Li^6 at 2.18 and 3.56 Mev, in Be^9 at 2.43 Mev, and in C^{12} at 15.1 Mev, were measured and analyzed by a virtual photon theory to give values of $(4^{+3}_{-1.5}) \times 10^{-4}$, 6.2 ± 0.6, 0.13 ± 0.03, and (40^{+8}_{-6}) ev for the values of their respective radiation widths to the ground states. Other well-defined peaks were observed at excitation energies of 16.9 Mev in Be, 11.6 Mev in Si, and 4.2 Mev in Pb. Broad peaks corresponding to the excitation of the giant resonance were observed in C, Si, and Pb, with maxima at 23, 20, and 15 Mev, respectively, and integrated cross sections of 75, 125, and 6500 Mev-mb, respectively. These cross sections are uncertain by a factor of approximately two because they depend on arbitrary methods used in subtracting the continuum of low-energy electrons and on arbitrary assumptions about nuclear form factors.

*On leave from Max-Planck-Institut für Chemie, Mainz, Germany.

†On leave from the Institut für Technische Kernphysik, Technische Hochschule, Darmstadt, Germany.

GAMMA RAYS FROM Ga^{65} DECAY AND HALF-LIFE OF THE 54-kev LEVEL OF Zn^{65}. L. S. August and J. F. Friichtenicht,* Radiation Division, U. S. Naval Research Laboratory, Washington, D. C. (Received August 1, 1960).

The energies and intensities of the gamma rays following the positron and electron capture decay of Ga^{65} have been determined by means of scintillation spectrometry techniques. At least twenty gamma rays de-excite proposed levels in Zn^{65} at the following energies (in kev): 53.95 ± 0.01, 115.13 ± 0.02, 206 ± 2, 773 ± 10 (?), 863 ± 10, 1040 ± 15, 1310 ± 20, 1480 ± 20, 1970 ± 40, and ~2300. A consistent cascade scheme for Zn^{65} is presented which incorporates all of the observed gamma transitions. The first excited state of Zn^{65} has been found to be isomeric with a half-life of 1.65 ± 0.05 μsec.

*Present address: Ramo-Wooldridge, Canoga Park, California.

HALF-LIVES OF At^{211} AND Bi^{207}. E. H. Appelman, Lawrence Radiation Laboratory, University of California, Berkeley, California, and Argonne National Laboratory, Argonne, Illinois (Received August 5, 1960).

Least-squares analysis of alpha-particle counting data has yielded a half-life of 7.214 ± 0.007 hr for At^{211}, in good agreement with previous results. From the gamma-ray activity remaining after the decay of an extremely active At^{211} preparation, the half-life of Bi^{207} has been found to be 38 ± 3 yr. This agrees with the highest of the values hitherto reported.

ABSOLUTE (d, α)-REACTION CROSS SECTIONS OF ZIRCONIUM, MOLYBDENUM, TITANIUM, AND SULFUR. Oswald U. Anders* and W. Wayne Meinke, Department of Chemistry, University of Michigan, Ann Arbor, Michigan (Received July 27, 1960).

Absolute cross sections for 7.8-Mev deuterons were measured for the following reactions: $Zr^{94}(d, \alpha)Y^{92}$*; $Zr^{92}(d, \alpha)Y^{90}$*; $Zr^{90}(d, \alpha)Y^{88}$*; $Mo^{97}(d, \alpha)Nb^{95}$*; $Mo^{97}(d, \alpha)Nb^{95m}$; $Mo^{98}(d, \alpha)Nb^{96}$; $Mo^{92}(d, \alpha)Nb^{90}$; $Ti^{46}(d, \alpha)Sc^{44}$; $Ti^{48}(d, \alpha)Sc^{46}$, and $S^{34}(d, \alpha)P^{32}$*. Excitation functions were also determined for the reactions indicated with an asterisk. Beam intensity was determined with a Faraday cage coupled to a current integrator able to measure the current with less than 1% error. The reaction products from the thin targets were chemically purified by carrier-free separations. The absolute disintegration rates were determined by 4π beta counting and coincidence gamma counting. Carrier-free tracer techniques were used to establish the yields of chemical purification steps.

*Present address: Dow Chemical Company, Midland, Michigan.

EVIDENCE FOR THE STATISTICAL MODEL OBTAINED FROM (p, n) SPECTRA OF THE ODD-EVEN ELEMENTS V, Mn, Co, Nb, Rh, AND In. Richard D. Albert, John D. Anderson, and Calvin Wong, Lawrence Radiation Laboratory, University of California, Livermore, California (Received August 1, 1960).

Spectra of neutrons emitted from (p, n) reactions have been measured for the essentially monoisotopic odd-even elements, V, Mn, Co, Nb, Rh, and In. The 7–8-Mev protons used were obtained from the Livermore 90-inch cyclotron. Neutron energy distributions were determined using time-of-flight techniques. Experimental results indicate agreement with the compound statistical model of the nucleus. Relative level densities obtained in this manner were found to increase with atomic weight as theoretically expected. Assuming a level density of the form $\exp[2(a\epsilon)^{1/2}]$, a values were obtained which varied as $a = A/13$.

POSSIBLE EFFECT OF $\bar{K}$-N REACTIONS ON K-N SCATTERING. Richard H. Capps,* Argonne National Laboratory, Argonne, Illinois, and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York (Received August 15, 1960).

Approximate P-wave dispersion relations for K-N scattering are considered, under the assumption that the $K\Lambda N$ and $K\Sigma N$ interactions are of the odd intrinsic-parity, Yukawa type. If the crossing terms (which involve the P-wave $\bar{K}$-N amplitudes) are neglected, the predictions of the equations are contradicted by the experimental evidence. However, the presence of virtual pion-hyperon states may enhance the $\bar{K}$-N amplitudes sufficiently that they play an important role in the equations. It is shown that

if the isotopic spin 1, $P_{3/2}$ $\overline{K}$-N absorptive amplitude is particularly strong, the contradiction with experiment may be removed. The possibility of even-parity K-baryon reactions is discussed briefly.

*Present address: Northwestern University, Evanston, Illinois.

DECAY $\Sigma^0 \to \Lambda^0 + e^+ + e^-$ AND THE Σ^0 - Λ^0 RELATIVE PARITY. Nina Byers, Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California, and Hugh Burkhardt, Department of Physics, California Institute of Technology, Pasadena, California (Received July 14, 1960).

The Σ^0 - Λ^0 relative parity may be measured by observing correlation of polarizations in the process $\Sigma^0 \to \Lambda^0 + \gamma$. Internal conversion of the photon into an electron pair (Dalitz pair) serves as an analyzer which selects polarized photons. Theoretical results are presented which show that the Dalitz-pair decay mode of polarized Σ^0's may be used to measure the Σ^0 - Λ^0 relative parity.

PHASE-PARAMETER REPRESENTATION OF PROTON-PROTON SCATTERING FROM 9.7 TO 345 Mev. G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Yale University, New Haven, Connecticut (Received August 4, 1960).

Results of gradient searches for phase parameters by means of an IBM-704 machine representing proton-proton scattering are reported on and the procedure used is described. The analysis made use of most available measurements. The number of "measurements" used by the machine was 541. In some cases each "measurement" was a composite of two or more measurements found in the literature. The fits join smoothly to the $^1S_0$ phase-shift energy curve below 9.7 Mev. A family of fits has been obtained employing as a starting point the extended source + spin-orbit potential representation below 150 Mev extrapolated to one or another of the several Stapp, Ypsilantis, Metropolis fits at 312 Mev. Other searches started out with phase parameters corresponding to the Gammel-Thaler phenomenological potential. Evidence is presented to the effect that the better fits of both families are essentially the same. Error limits of phase parameters derived are estimated by various procedures. Some of the accumulated evidence for the applicability of the one-pion exchange potential (OPEP) to the calculation of phase parameters for the higher L and J is described and the basis for previously published inferences regarding the spatial extension within which the major part of the potential is the OPEP is illustrated. In the case of the best fit, referred to as YLAM, tests indicating the existence of potentials for separate J have been made showing therefore that there is no serious question regarding the energy rates of change of logarithmic derivatives of radial wave functions being negative. A brief discussion of the relationship of this requirement to the meson theory of nuclear forces is included.

SEARCH FOR Σ^- HYPERFRAGMENTS. R. G. Ammar, N. Crayton, K. P. Jain, R. Levi Setti, J. E. Mott, P. E. Schlein,* O. Skjeggestad, and P. K. Srivastava, Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois (Received August 9, 1960).

A systematic search has been undertaken for possible Σ^- compounds produced by K^- captures in nuclear emulsion. Mass determinations were performed on (a) 15 decays in flight into a charged pion and (b) 84 events resulting in capture configurations; the number of these events produced in single-nucleon K^- captures is estimated to be >32 but <71. All events of type (a) and all but 4 of type (b) are consistent with the interpretation as Σ^-. The 4 events are discussed in detail and it is concluded that, although the $(\Sigma^- n)$ interpretation cannot be ruled out for 3 of them, in no case is the identification unequivocal.

*Now at The Johns Hopkins University, Baltimore, Maryland.

STRUCTURE OF THE S MATRIX IN THE PRESENCE OF A BOUND STATE. Smio Tani,* Department of Physics, Washington University, Saint Louis, Missouri (Received August 12, 1960).

It is shown that the phase factor associated with the "orthogonality phase shift" due to a bound state should be factored out of the S ma-

trix. A crucial test of this statement is found in a study of the final-state interaction of an inelastic process which ends in a channel involving the bound state. If we assume that the sum of the Born series for the S matrix gives a right answer after we separate the effect of the bound state in terms of the orthogonality phase shift, agreement with Watson's result obtains only when the S matrix has the factored structure.

*On leave of absence from Tokyo University of Education, Tokyo, Japan. Address after September, 1960: Physics Department, New York University, University Heights, New York, New York.

K_2^0 AND THE EQUIVALENCE PRINCIPLE. Myron L. Good, University of Wisconsin, Madison, Wisconsin (Received July 18, 1960).

It is shown that the existence of the long-lived neutral K meson, and the absence of its decay into two pions, establishes that the gravitational masses of the K^0 and $\overline{K}^0$ are equal to a few parts in 10^{-10} of the K inertial mass. This is of interest since the $\overline{K}^0$ is the antiparticle of the K^0, and is not identical with the K^0. The gravitational mass of such a nonidentical antiparticle has never been directly measured.

Also, the $\overline{K}^0$ has opposite strangeness to the K^0. Thus, the argument rules out any linear dependence of the gravitational mass on the strangeness quantum number, a point on which all previous experiments say nothing.

These observations are in accord with, and serve as a confirmation of, the equivalence principle of Einstein.

THEORY OF LOW-ENERGY NUCLEON-NUCLEON SCATTERING. M. L. Goldberger, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey, M. T. Grisaru, University of Illinois, Urbana, Illinois, S. W. MacDowell,* Palmer Physical Laboratory, Princeton University, Princeton, New Jersey, and D. Y. Wong, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received August 15, 1960).

The two-nucleon problem is discussed from the standpoint of the double dispersion relations. The analytic structure of partial wave amplitudes is completely analyzed. This is greatly facilitated by the use of the Jacob-Wick helicity amplitudes. The program of generating a set of dynamical equations by use of the unitarity condition is carried out. In the present approximation only one- and two-pion exchanges are considered; the resulting system of equations should be adequate for energies below about 170 Mev. The problem of computing the deuteron parameters is discussed. The general structure of the more complicated nucleon-antinucleon system is briefly treated.

*On leave of absence from Centro Brasileiro de Pesquisas Fisica, Rio de Janeiro, Brazil.

WEAK-COUPLING CURRENTS AND SYMMETRIES OF STRONG INTERACTIONS. R. E. Behrends,* Institute for Advanced Study, Princeton, New Jersey, and A. Sirlin,† New York University, Washington Square, New York, New York (Received July 29, 1960).

The general isotopic properties of bilinear currents which will lead to the $|\Delta S| \leq 1$ and $|\Delta I| = \frac{1}{2}$ rules for weak decay processes are examined. The latter rule is re-expressed in terms of an equivalent mathematical statement which permits one to obtain the usual predictions in a simple manner. In general, when the strangeness-conserving part of such a current is an isotopic vector, the strangeness-changing part can be a linear combination of $I = \frac{1}{2}$ and $I = \frac{3}{2}$ currents. The existence of an $I = \frac{3}{2}$ current could be established by experiments on the decays $K \to \pi +$ leptons, or on high-energy neutrino capture, $\nu + N \to \mu + \Sigma$. Experiments on K_{e4} decays could test the bilinearity of the current.

The assumption that the vector part of such a current, both strangeness changing and nonchanging, is quasi-conserved (i.e., neglecting certain mass differences) in the presence of the strong interactions fixes the specific form of the current and further implies symmetries for the strong couplings. The various transformations which leave invariant a Yukawa-type strong interaction as well as their associated currents are found. A new possible symmetry group of the strong interactions is examined: a 14-parameter group usually denoted as $G2$. In the presence of both π and K couplings, it is found that $I = \frac{1}{2}$ and $\frac{3}{2}$ currents are quasi-conserved when the strong Lagrangian has a 7-dimensional rotational symmetry, while for the $I = \frac{3}{2}$ alone, the symmetry required is $G2$. In the presence of only π-baryon couplings, only $I = \frac{1}{2}$ currents can be

quasi-conserved. Certain predictions for the K_{e3} and K_{e4} modes of decay and for $\Sigma^- \to n + e^- + \nu$ follow from the weak currents determined in this way.

*Present address: Physics Department, University of Pennsylvania, Philadelphia, Pennsylvania.

†Presently on leave from New York University to CERN, Geneva, Switzerland.

RENORMALIZATION CONSTANTS IN QUANTUM ELECTRODYNAMICS.

Paul C. DeCelles, The Johns Hopkins University, Baltimore, Maryland (Received June 10, 1960).

The electron self-energy and wave-function renormalization constant are calculated with the use of the technique of dispersion relations. In order to do this, exact representations of these renormalization constants are obtained in terms of matrix elements of Heisenberg field operators. The matrix element $\langle e\gamma|\bar{\psi}|0\rangle$ is studied in detail and its contribution to the renormalization constants determined. In retaining only the contributions due to $\langle e|\bar{\psi}|0\rangle$ and $\langle e\gamma|\bar{\psi}|0\rangle$, an infinity of other matrix elements is ignored. A test of the validity of this approximation is discussed. Within the limits of the approximations, the self-energy and wave-function renormalization constants are shown to have a logarithmic, ultraviolet divergence. The wave-function renormalization constant also has an infrared divergence.

A TWO-DIMENSIONAL RELATIVISTIC FIELD THEORY.

P. Federbush, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received August 4, 1960).

A particular two-dimensional relativistic field theory is considered. In some limit as the masses of the theory go to zero it approaches the Thirring model. By means of a formal transformation of the field operators the Hamiltonian is reduced to that of a free field. An improved perturbation expansion can be written down, necessitating only wave-function renormalization, and it appears that the renormalized theory is consistent. The S matrix can be exhibited exactly, and though it leads to no physical scattering, it is not equivalent to the unit matrix. Finally the renormalized current operator is displayed as a suitable limit of products of the renormalized field operators. The form of the result clearly separates the consistency problem in quantum electrodynamics from that of the "photon mass."